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THOMSON SCATTERING AND PLASMA OSCILLATIONS

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Abstract

A direct calculation shows that Thomson scattering -- on the microscopic level -- manifests itself as plasma oscillations -- on the macroscopic level.

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An electromagnetic wave passing through a medium can be described on either of two levels: on the microscopic level, Thomson scattering of the electromagnetic wave, by the electrons in the medium, acts to shift the phase of the wave; on the macroscopic level, the phase of the wave is fixed by the index of refraction of the medium, which is derived by considering the plasma oscillations of the electrons. Equating the results of the two points of view shows that Thomson scattering, on the microscopic level, is manifest, on the macroscopic level, as plasma oscillations.

To show this, consider a wave, moving in the z direction, and scattered by a thin slab of thickness Δ. We suppose n is the density of scatterers in the slab and $f(\theta)$ is the scattered amplitude in the direction θ. The total wave at a distance z from the slab is

$$\psi(z) = e^{ikz} + \int_0^\infty dr\Delta\, 2\pi r n f(\theta)\, \frac{e^{ikR}}{R} \quad ,$$

where $R = \sqrt{r^2+z^2}$, k is the wave number ($k=2\pi/\lambda$) of the wave, and r measures distance, in the radial direction, from the z–axis.

Since $RdR = rdr$, this may be written

$$\psi(z) = e^{ikz} + 2\pi n\Delta \int_z^\infty dR f(\theta)\, e^{ikR} \quad .$$

According to the Fresnel Theorem, the integral equals 1/2 the amplitude from the first Fresnel zone; that is, 1/2 the integral evaluated from z to $z+\frac{\lambda}{2}$:

$$\psi(z) = e^{ikz} + 2\pi\Delta f(0) \int_z^{z+\lambda/2} \frac{dR}{2}\, e^{ikR}$$

$$= e^{ikz}\left(1 + i\,\frac{2\pi n\Delta f(0)}{k}\right) \quad .$$

On the other hand, if the index of refraction of the slab is η,

$$\psi(z) = e^{ik(\eta-1)\Delta+ikz} \sim e^{ikz}\,(1 + ik(\eta-1)\Delta) \quad .$$

Equating these two expressions, supposing η close to 1, and using the plasma oscillation value for η^2, leads to ($r_o = e^2/mc^2$)

$$\frac{2\pi n f(0)}{k^2} = \eta-1 \sim \frac{1}{2}\,(\eta^2-1) = -\frac{2\pi n r_o}{k^2} \quad ,$$

from which we find

$$f(0) = -r_o \quad .$$

Thus, the total scattering cross-section is

$$4\pi r_o^2 \quad .$$

This calculation is for scalar scattering. For unpolarized, vector scattering, the total cross-section is given by the usual average:

$$\sigma_{tot} = 4\pi r_o^2 < \frac{1}{2}\,(1 + \cos^2\theta)> = \frac{8\pi r_o^2}{3} \quad ,$$

which is the familiar expression for Thomson scattering, thereby substantiating the claim in our initial paragraph.

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