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INSTABILITIES AND TURBULENCE ON THE LATTICE

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ABSTRACT

Building a quadratic map with a characterization parameter λ on the lattice with an interaction scheme between the nearest sites of strength g, we have found rich behaviors in the λ–g parametric space. We identify the Kolmogorov and Feigenbaum regimes in this space: they represent the g and λ axes respectively. Away from the axes, there exists a domain R where patterns are formed. Non–commensurate frequencies, entrainment and phase locking manifest in region R. As we journey down in g at a fixed λ, Kolmogorov type Fourier spectra are found.

In this scheme, we recovered the Feigenbaum predictability of onset of turbulence and discover a different route to turbulence: the route to the Kolmogorov spectrum.

When this method is applied to a pipe flow problem, laminar and turbulent profiles are generated with surprisingly detailed features: the dynamics of the growth of the sub–laminar boundary layers, the transition, and the turbulent regions are found to be intimately dependent on g.

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In systems far from equilibrium, microscopic behavior will eventually emerge as macroscopic order and instabilities. These instabilities will eventually break down into seemingly chaotic motion. The order is still retained somehow in this chaotic state, and this is turbulence. In these systems far from equilibrium, microscopic fluctuations do not have to be comfined into small scales. In simple cases these fluctuations can be averaged but when we take a larger sample, when we measure them with a probe much larger than the microscopic fluctuation scale, the averaged measured quantities satisfy classical continuum equations. In hydrodynamics, microscopic fluctuations average out and classical hydrodynamics equations emerge.

However, when the systems, for example, manifest criticality of second order, fluctuations persist in all scales from microscopic to macroscopic wavelengths. There is no classical continuum equation that can be used to describe the outcome of the chaotic behavior, and somehow novel means have to be invented to handle this type of problem.

In fully developed turbulence, instabilities are somehow set in, eddies of large scale are formed. These eddies will eventually break up into smaller eddies and smaller yet to a point where dissipative energy due to viscosity sets in, and the eddies will end their long, chaotic but orderly journey. In atmospheric turbulence, the scale of these eddies vary from thousands of kilometers down to fractions of millimeters. The physics of many scales of fluctuation is what makes these problems difficult. Progress has been made in the field of elementary particle physics and critical phenomena, but turbulence has remained a mystery, until recent works of Feigenbaum, where the manifestations of the onset of turbulence has been successfully explained.[1]

The Feigenbaum route to turbulence is via period doubling bifurcation. As the parameter of the recursion relation increases, the system goes into chaos, but this is not fully developed turbulence. Feigenbaum turbulence manifests a flat Fourier spectrum, while fully developed turbulence manifests Kolmogorov-like spectrum of the form $k^{-5/3}$ where k is the wave number.

(Experimentally, the power varies as $-(\frac{5+\mu}{3})$ where μ varies from 0 to .5, depending on experiments.) Furthermore, experimentally there exist routes to turbulence that do not manifest period doubling. They exhibit more complicated behaviors;[2] non-commensurate frequencies are found in this route, together with frequencies entrainment and phase locking. In this theory, we start from a microscopic description on lattice sites and build a recursion relation. For a given driving parameter, the coupling between sites undergoes criticality, and instabilities are formed as macroscopic order. By decreasing this coupling, instabilities will break up in a fashion described by a renormalization class and fully developed turbulence appears.

Besides retaining Feigenbaum's success, we obtain pattern formation (eddies) and describe the orderly decay of the instabilities to turbulence. All the experimental results discussed above, i.e., incommensurate frequencies, entrainment and phase locking are obtained, and most surprisingly a Kolmogorov spectrum appears as the coupling between sites decreases further. The intermittency μ is trivially obtained as a consequence of moving away from the classical Kolmogorov picture where $\mu = 0$ and μ is found to be saturated at about $\frac{1}{2}$ at the bifurcation point.

Putting turbulence on the lattice means we treat turbulence as a many degrees of freedom problem. In this theory, it will be shown that a <u>class</u> of functions manifests all the properties described above. Let's start with a recursion relation which describes the space and the time evolution of a microscopic fluctuation U (in one dimension first):

$$U_{n+1}^{m} = Q(U_n^m) + \frac{g}{2}(U_n^{m+1} + U_n^{m-1} - 2U_n^m). \quad (1)$$

$Q(U_n^m)$ represents a class of functions which is quadratic; U is the fluctuation; n stands for discrete time; m stands for discrete space and g is the coupling between immediate neighbors.

As in the problem of the onset of turbulence of Feigenbaum, $Q(U_n^m)$ is required to be quadratic but the proper form does <u>not</u> have to be specified. The following forms have been used:

$$Q(U_n^m) = \lambda U_n^m (1-U_n^m)$$

and

$$Q(U_n^m) = \lambda \sin(\pi U_n^m)$$

The same results are obtained using either one relation. In the following study, the logistic form is used for its simplicity. By going into a lattice, the parameter space is changed into a larger one involving λ and g. Fig. 1 defines the physics of this parametric space. Besides identifying $g = 0$ as the Feigenbaum regime, we discovered that the regime $\lambda=1$ (for the logistic map) is the classical, Kolmogorov regime.

We found that there exists a regime R where the Fourier spectrum of the U_n exhibits resonances, and as g is decreased to region M, down to Feigenbaum regime a Kolgomorov-like spectrum appears. At <u>any</u> λ outside the classical regime, as the coupling g decreases, <u>we form instabilities and watch them decay into fully developed turbulence</u>.

To illustrate this sequence, we used $Q(U_n^m) = \lambda U_n^m (1-U_n^m)$ in Eq. 1 for simplicity. We studied this problem on the stability branch, i.e., λ between 1 and 3. The reader should be reminded that Feigenbaum's onset to turbulence manifests at the accumulation $\lambda_\infty = 3.57$. Starting with a string of m's at random values of U's, if λ's between 1 and 3, and $g = 0$, then as time n goes on, the recursion relation should bring U's to a common fixed point, and subsequent values of U's in the array will be the same fixed point values.

But surprisingly this is not the case when g is turned on. Communication between nearest neighbor sites eternally confines the recursion relation into a harmonious transient to the fixed point, and in the region R along this this seemingly chaotic transient journey to the fixed point, resonances are formed. This is the result of the renormalization scheme. We have built a lattice of 2000 sites looped back to make a continuous boundary condition and have evolved U_n up to 2^{12} times before Fourier transforming U_n to obtain P(f) where f is the frequency. Figures 2a, 2b show the frequency spectra at $\lambda=1.5$, $g=.918$, and $g=.910$ respectively. This spectra was observed experimentally.[2]

One of the most interesting features of the experimental data is that there appears a second or third mode of frequencies. The appearance of non-commensurate frequencies, entrainment and locking as observed by Libchaber et al., and other workers[2] is a consequence of the conspiracies of many frequencies on the lattice. Figs. 3a, 3b, and 3c are obtained at $\lambda = 1.5$ and $g = .9100$, .9075, and .9025, respectively.

As we journey out of the resonance region R, to the instability breaking region M, the spectrum becomes smooth and a Kolmogorov-like spectrum appears and continues to have an invariant slope up to the Feigenbaum regime (Figs. 4a, 4b).

Note that these behaviors occur in the stability branch of the map before the bifurcation region.

We will now discuss the classical regime. Take a nonlinear evolution equation:

$$U = aU - bU^2 \tag{2}$$

Solving it exactly yields uninteresting results. The corresponding recursion relation ($\tau = \frac{t}{m}$) is:

$$U_{n+1} = U_n + (aU_n - bU_n^2)\,\tau$$

With $\lambda = 1+a\tau$ and with a scale transformation: $X = \frac{b\tau}{\lambda} U = \gamma U$, one gets the logistic map[8].

$$X_{n+1} = \lambda X_n [1 - X_n] \tag{3}$$

In the systems far from equilibrium, the Poincaré time τ is proportional to the fluctuation of the system as shown by Ehrenfest and Smoluchowski. A continuous solution to the equation 2 (the Landau equation) means $\tau = 0$. It also means that the system has no fluctuations. But if we want to build a microscopic theory, we should be prepared to incorporate fluctuations at the onset because at the critical point, fluctuations grow into all scales. In equation 3, if a is fixed, λ increases as τ increases. Thus at $\lambda \approx 1$, the classical picture is preserved because $\tau = 0$ in this case. We found that at $\lambda = 1.05$, the spectrum is identically $-5/3$ in power (Fig. 4a) and λ increases, the power increases in magnitude (Fig. 4b).

Fig. 4a is identical to experimental spectra of streamwise and lateral velocity fluctuation for an axis-symmetric jet.[4] The deviation from Kolmogorov spectrum (-5/3) to a spectrum with intermittency μ is shown in Fig. 4b. From the eddy decay model (β-model for example), the self-similar constraint forces the spectrum to have the power $-(\frac{5+\mu}{3})$. From this research, we found $\mu=0$ at $\lambda=1$ and $\mu\approx.5$ at $\lambda=2.9$. We have shown that $\lambda=1$ is the classical Kolmogorov limit, and thus a non-zero value of μ is an indication of how far we depart from the classical limit. The readers are reminded that, experimentally, μ varies between 0 and .5.

Finally, we want to elaborate on the fact that in all systems that manifest bifurcation, Feigenbaum δ is about 4.6. This corresponds to a quadratic nature of the map. We have used $Q(U) = \lambda \sin(\pi U)$, for example, and found the results to be independent of Q(U) as long as Q(U) is quadratic. Equations of the form of eq. 2 with multiplicative fluctuations manifest second order phase transition while eq. 2 with a cubic nonlinearity manifest first order phase transition. The fact these systems are of quadratic in nature may indicate that turbulence belongs to the critical phenomenon class: second order phase transition. This is the reason for the growth of fluctuations to all scales. To illustrate the necessity for a quadratic form, we have used Q(U) in the form of a random generator and found that the spectrum is flat.

Furthermore, for a lattice of 2000 sites and with 2^{12} iterations, at λ=3.5982 and g=.001, the bifurcation structures generated from this work, are identical to Feigenbaum's structures.

So far, we used a periodic boundary condition for the one-dimensional lattice. But to produce the velocity profiles in a pipe, for example, we should set the U's at both ends to zero. In the classical limit, $\lambda = 1$, we found a parabolic profile at g = .90. This corresponds to the laminar profile. However, as g decreases, a flat velocity spectrum was generated. This corresponds to a turbulent profile. Detailed investigations at the regime near the boundary (U = 0) show that the growth from U = 0 to the constant U of the flat spectrum is indeed a logarithmic growth. As g decreases from the laminar value, ($\lambda = 1$, g = .9), the sub-laminar boundary layer region closest to the pipe boundaries gradually disappears and at g close to the Feigenbaum axis, the velocity growth is found to be exactly logarithmic. These detailed profiles generated by the map, which is in agreement with experimental profiles in pipe flow, lends support to the practical validity of this theory.

By putting a quadratic map on a lattice with direct coupling between the sites, the transient route to fixed points manifests rich structures and the decay of these structures is of Kolmogorov type.

In a two dimensional case with triangular lattice, we have found that, starting from random values of U's at the initial lattice of 60 x 60 x 60 sites, hexagonal patterns, mimicking Benard cells are formed. Most surprisingly, they are formed only after few iterations. Pattern formations and detailed results of this work can be found in a longer versions of this paper[5].

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FIGURE CAPTIONS

Fig. 1: The parametric space of the parameter λ of the quadratic function and the coupling g of the nearest neighbor sites. The description of different regimes is found in the text. Structures are formed in region R and Kolmogorov type spectra are formed in region M.

Fig. 2: The Fourier spectra P(f) of the recursion relation with $\lambda = 1.5$ and $g = .918$ (2a) and $g = .910$ (2b).

Fig. 3: The Fourier spectra P(f) of the recursion relation with $\lambda = 1.5$ and $g = .9100$ (3a), $g = .9075$ (3b) and $g = .9025$ (3c). The non–commensurate frequencies growth, entrainment and finally phase locking phenomena are shown.

Fig. 4: The Fourier spectra (Pf) of the recursion relation. Fig. 4a is generated with $\lambda = 1.05$ and $g = .5$. With λ near 1, a Kolmogorov spectrum is observed. The slope within accuracy of the plot is –5/3. In 4b: a,b($\lambda = 1.5$) and c,d($\lambda = 2.9$) with different g's for each case.

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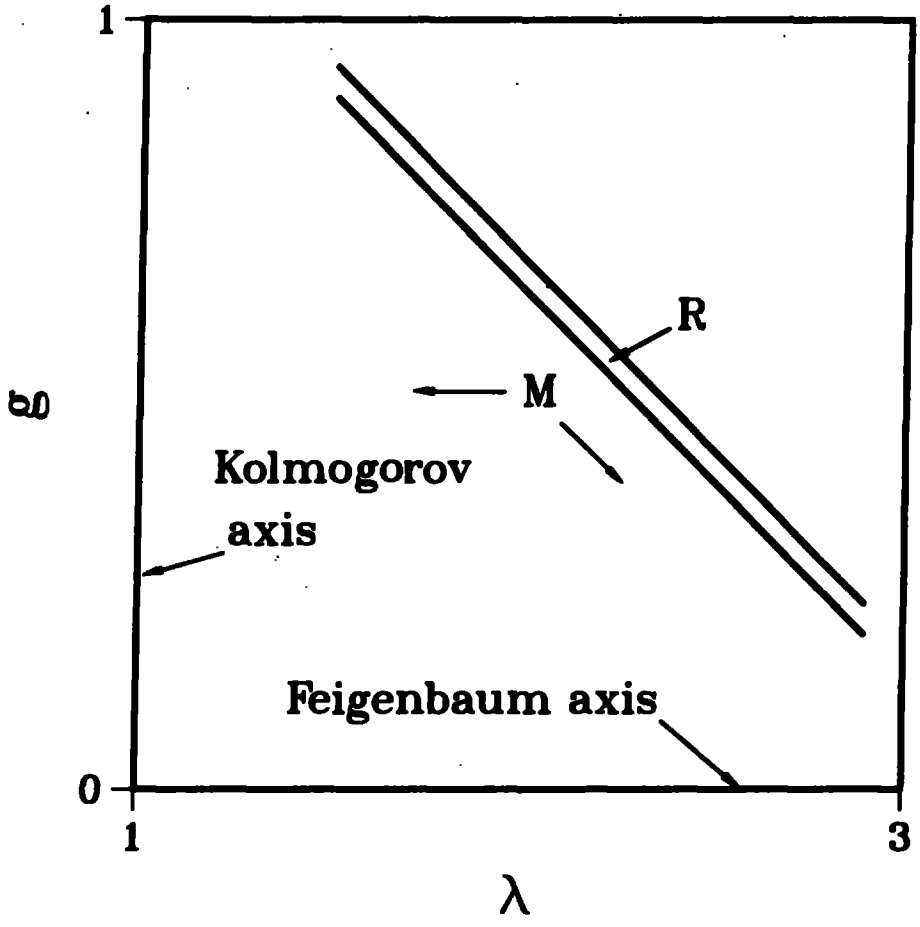
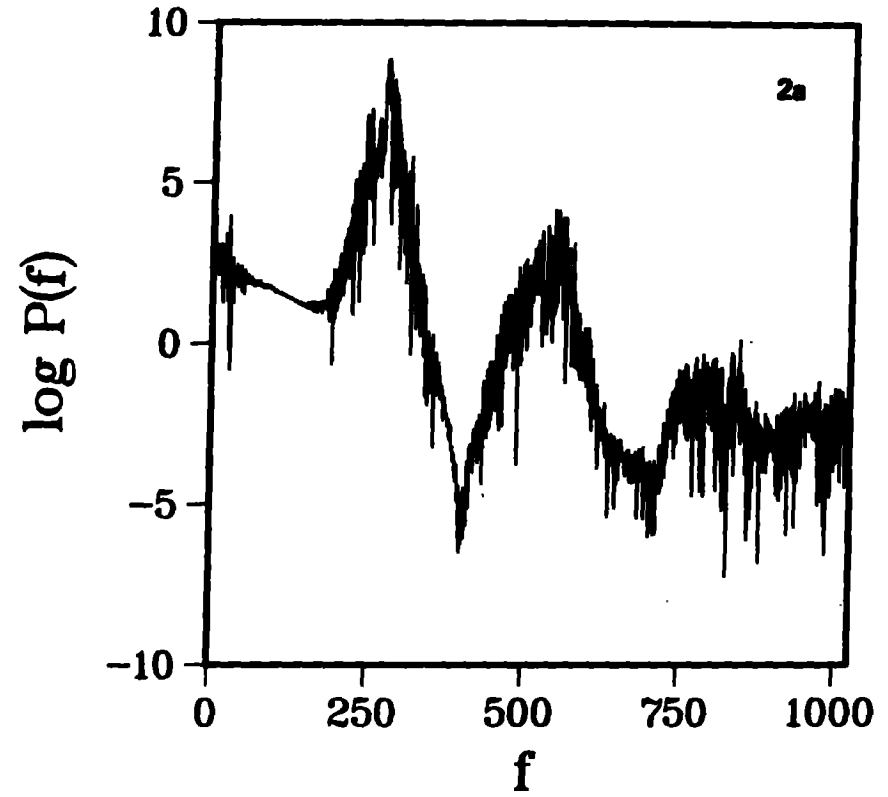


Figure 1


2a
log P(f)
10
5
0
−5
−10
0
250
500
750
1000
f

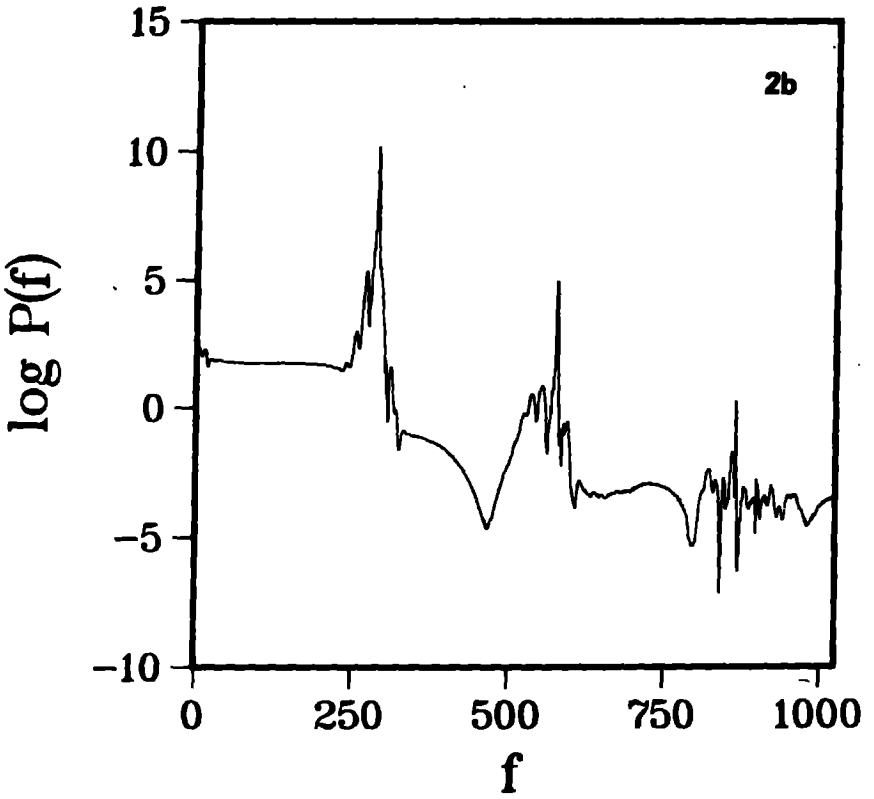

2b
log P(f)
15
10
5
0
−5
−10
0
250
500
750
1000
f

Figure 2

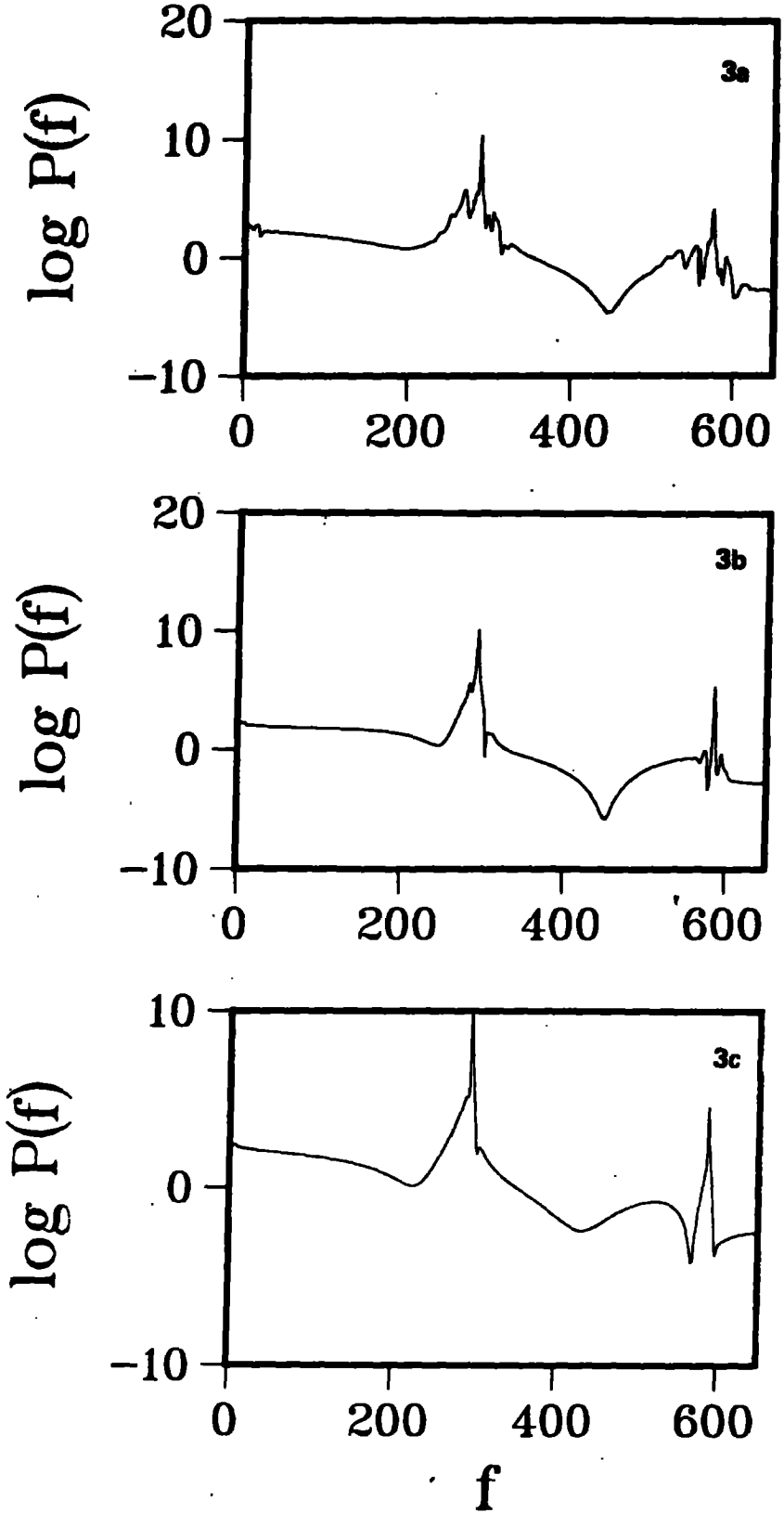


Figure 3

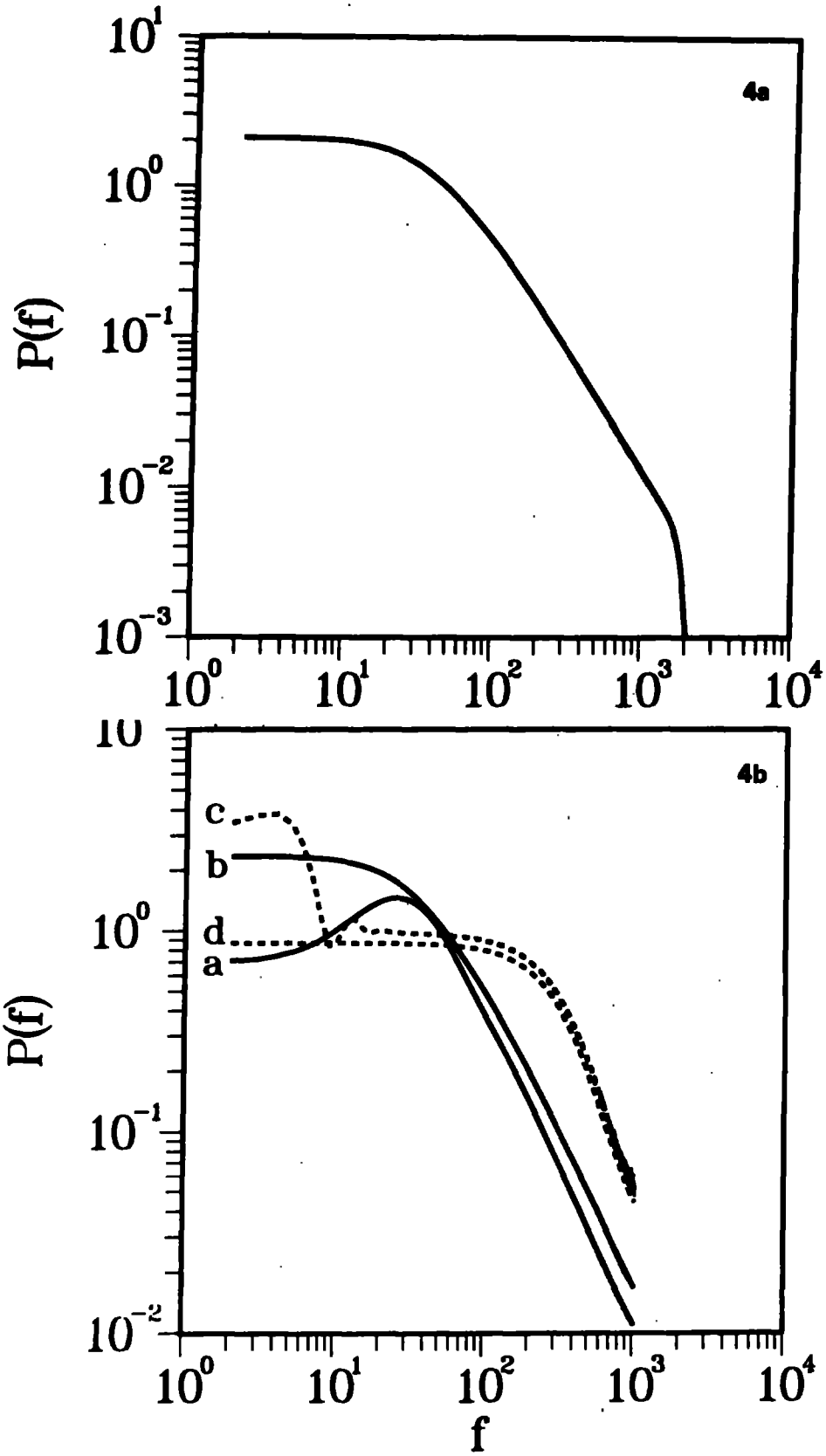

4a
P(f)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
4b
c
b
d
a
P(f)
10^{0}
10^{-1}
10^{-2}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
f

Figure 4