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PHYSICS ISSUES IN MIRROR AND TANDEM MIRROR SYSTEMS*

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ABSTRACT

Over the years the study of the confinement of high temperature plasma in magnetic mirror systems has presented researchers with many unusual physics problems. Many of these issues are by now understood theoretically and documented experimentally. With the advent of the tandem mirror idea, some new issues have emerged and are now under intensive study. These include: (1) the generation and control of ambipolar confining potentials and their effect on axial confinement and, (2) the combined influence of nonaxisymmetric magnetic fields (used to ensure MHD stability) and electric and magnetic particle drifts on radial transport. Physics considerations associated with these two categories of issues will be reviewed, including concepts for the control of radial transport, under study or proposed.

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I. INTRODUCTION

During the three plus decades over which the magnetic mirror approach to the confinement of hot plasma has been under investigation, many important physics issues have had to be addressed. In the course of the research most of these have by now been successfully resolved. These include the issues of MHD equilibrium and stability and the elucidation and control of high frequency instabilities. Theory predicts and experiment confirms the existence of high beta MHD stable equilibria in mirror systems with plasmas that are also at the same time virtually free of high frequency instabilities. With the apparent resolution of these issues, emphasis in the research is shifting to more specific quantitative ones: What are the particle and energy confinement times as dictated by axially and radially directed particle transport?

With the introduction of the tandem mirror idea, a major improvement over conventional mirror systems was made in axial confinement, predicted by theory and confirmed by experiment. The new issue (new to mirror research, that is) is that of the competition between radially- and axially-directed particle losses. In this paper we will review the present status of the theory of these transport processes, particularly as they are influenced by the geometry of the confining magnetic field and by the ambipolar potential and the diamagnetic currents of the confined plasma.

II. REVIEW OF PRINCIPLES OF MIRROR CONFINEMENT

The confinement of charged particles in an open-ended magnetic field such as a mirror machine is rooted in the existence of adiabatic invariants associated with nearly-periodic motions of trapped particles. The nearly-periodic motions here involved are threefold:

- Gyration of the particles, around their guiding center, at the cyclotron frequency, ω_c.
- Oscillation of the trapped particles, between reflection points at the mirrors, at the bounce frequency, ω_b.
- Azimuthal drifting of the particles around the magnetic axis of the system, at the drift frequency ω_d.

As understood from classical mechanics, associated with each such nearly-periodic motion there exists an "adiabatic invariant", the near-constancy of which is assured provided the parameters of the system are

sufficiently slowly varying on the time scale of that particular periodic motion.

Typically, in a mirror system the three frequencies are well separated, with $\omega_c >> \omega_b >> \omega_d$. The corresponding invariants are, (1) the magnetic moment, $\mu = \omega_\perp/B$, associated with the cyclotronic motion, (2) The longitudinal action, $J = \oint v_{\parallel} dt$, associated with the bouncing motion, and (3) Φ, the flux enclosed by the particle centers as they drift around the magnetic axis. Constancy of μ ensures mirror reflection, i.e., that particles will be confined axially; constancy of J ensures that the particles will remain on closed drift surfaces within the system, i.e., that they will be confined radially. Confinement suffers when the system or plasma parameters are such as to perturb either of these invariants.

Conventional mirror systems, that is, those with simple mirror cells, rely on the constancy of μ to ensure axial confinement of plasma ions. Electrons are in this case in the main confined axially by the naturally-arising positive ambipolar potential of the plasma. Tandem mirror systems employ designed-in variations in this ambipolar potential to enhance the confinement of ions in a long central mirror cell. This cell is bounded at each end by short mirror cells that provide the electrostatic plugging action. Figure 1 is a schematic drawing of a tandem mirror system, in its simplest embodiment.

Although, as in the case of the tandem mirror, axial confining forces may be provided in part through electrostatic fields, these forces ultimately rely on the magnetic forces provided by the end mirrors, coupling to the magnetic moments of confined ions in the end cells. That is, ultimately axial confinement must depend on the axially-directed force, represented by the summation of the infinitesimal forces $dF = -\mu\nabla B$ operating on the confined particles. The first concern in any mirror system, conventional or tandem, is therefore with the constancy of μ for the ions.

Perturbations in μ can arise from three general sources:

- ion-ion and ion-electron collisions
- high frequency ($\omega \approx \omega_c$ or higher) instabilities
- spatial gradients in the magnetic field.

The first of these is a "given"; collisions can only be minimized by employing high plasma temperatures, with ion-ion collisional times scaling up with ion energy according to the typical "Spitzer relaxation time". For deuterons this time is given by the expression:

$$n\tau_{ii} \approx 2.2 \times 10^{10}\, \bar{E}_{keV}^{3/2} \quad cm^{-3}\ sec. \tag{1}$$

The second perturbing influence, high frequency instabilities, must be reduced to an acceptable level, defined later, to allow good confinement. This reduction is effected by control of the plasma distribution functions and spatial gradients in an optimized manner, later described.

The third perturbing influence on μ is the presence of spatial gradients in the magnetic field, as measured on a scale-size of the gyroradius of the ion. As the ions move back and forth between the mirrors, their cyclotron frequency changes with the changing local value of the magnetic field. Too rapid variations (i.e., two steep axial magnetic gradients) will perturb the magnetic moment. An insightful way to look on this problem, one that also permits its analysis, is to view these perturbations as arising from parametric resonances between the cyclotronic motion and higher order harmonics of the bounce frequency. The upshot of the analysis is to show that the fluctuations in μ that arise from these resonances decrease exponentially with the reciprocal of the "adiabaticity parameter", $\epsilon = a_i/L_m$, where L_m is a characteristic length of the magnetic field gradient. Thus $\delta\mu/\mu \sim \exp(-1/\epsilon)$. Though it represents a design constraint, nonadiabaticity can always be made to be sufficiently small in a large enough system or at high enough fields (small a_i).

Just as μ controls axial confinement and transport in mirror systems, J controls radial confinement and radial transport. Fixed μ and J define drift surfaces on which particles are constrained to remain as they bounce back and forth between the mirrors and slowly precess in azimuth. On a slow time scale collisions will alter μ and J and thus allow particles to diffuse to adjacent drift surfaces. Instabilities with Fourier components at the ion cyclotron or ion bounce frequencies can produce the same effect and must therefore be adequately suppressed. Of greatest concern, however, are perturbations of J of geometric origin. That is, perturbations that arise from resonating interactions involving harmonics of the drift frequencies. These can arise from azimuthal asymmetries of the magnetic field (or the plasma). These harmonics may couple to the bounce motion, thus perturbing J and therefore leading to unacceptably rapid radial transport. This problem arises particularly in tandem mirror systems in which nonaxisymmetric mirror fields are used in order to achieve MHD stability. The solution of such problems requires care in the design and in the control of the plasma parameters in tandem mirror systems. The topic is one that is under intensive

study at the present time in mirror research. Some of the results of these studies will be reviewed in this report.

To summarize these introductory remarks, mirror confinement differs fundamentally from confinement in toroidal systems in that both axial and radial confinement and transport must be considered. Two adiabatic invariants operate to ensure good confinement in both directions (together with associated ambipolar potentials in the case of the tandem mirror). The most serious threat to axial confinement arises from high frequency instabilities coupling to the cyclotronic motion of the ions. Ways have been found to suppress these instabilities. Radial confinement is controlled by the second invariant, J, associated with the bouncing motion of the ions between the mirrors. This invariant can be perturbed, and radial transport thus enhanced, by resonant coupling between the ion bounce frequencies and harmonics of the azimuthal drift frequency. These harmonics can arise from azimuthal asymmetries in the magnetic field. Such asymmetries are inherent in some tandem mirror systems, having been introduced in the design in order to achieve MHD stability.

III. AXIAL CONFINEMENT: TRANSPORT THROUGH THE MIRRORS

In this section we will discuss some critical issues in the control of mirror losses, including the role of the ambipolar potential, the effects on confinement of fluctuating fields arising from high frequency instabilities, and the means that have been developed to suppress these instabilities.

A. Collisional Losses

As noted in the previous section, in the absence of other perturbing influences ion-ion collisions set an upper bound on the confinement time of plasma in a conventional mirror machine. This problem, one of the earliest examined in mirror research, has been thoroughly studied through the use of the Fokker-Planck equation, solved with computer programs of increasing degrees of realism (and complexity). An article by Killeen, Mirin, and Rensink /1/ describes some of the larger such codes and reviews earlier work. The F-P code results on axial confinement times can be summarized succinctly in Eq. (2) (of use for later comparisons) giving the confinement time parameter $n\tau$ (for a deuteron-electron plasma):

$$n\tau = 2.3 \times 10^{10}\, \bar{E}^{3/2} \log_{10} (R_m) \qquad (2)$$

where $\overline{E}$ is the mean ion energy (from neutral beam injection) and R_m is the "mirror ratio" (ratio of magnetic field at the mirror to that at the minimum).

B. Fluctuation-Induced Losses

Equation (2) can be used as a standard against which to compare other effects, such as the effects of fluctuations arising from high frequency instabilities. These kinds of problems can be treated by the methods of quasilinear theory, which has been shown to be reasonably reliable in such situations. As derived, for example, by Berk and Stewart /2/, the effects of a fluctuating potential, ϕ, associated with unstable waves having $k_{\parallel} << k_{\perp}$ and centered in the vicinity of the ion cyclotron frequency can be described by a diffusion coefficient in $v_{\perp}$:

$$D(v_{\perp}) = \frac{1}{M^2 v_{\perp}^2} [e\phi]^2 \left(\frac{\omega_{ci}^2}{\Delta\omega}\right) J_1^2(k_{\perp} a_i) \quad . \tag{3}$$

Here $\Delta\omega$ is the fluctuation correlation frequency, which can be taken to be of the order of the reciprocal of the ion bounce time, τ_b. The argument of the Bessel function, $k_{\perp} a_i$, is the product of the perpendicular wave number $k_{\perp}$ and the mean ion orbit radius, a_i. Typically $k_{\perp} a_i >> 1$.

From $D(v_{\perp})$ it is possible to deduce a characteristic diffusion time, t_d, that may be used as a rough estimate of the fluctuation-limited axial ion confinement time (in the same sense that the Spitzer relaxation time is such a measure). We have

$$<\Delta v_{\perp}>^2 = D(v_{\perp})\ \tau_d \tag{4}$$

and

$$\Delta W_{\perp} = M v_{\perp}\ \Delta v_{\perp} \ . \tag{5}$$

From these relations it follows from Eq. (3) that

$$\tau_d = \left\{\left[\frac{\Delta W_{\perp}}{e\phi}\right]^2 \left[\frac{1}{J_1^2(k_{\perp} a_i)}\right] \left[\frac{1}{\omega_{ci}\tau_b}\right]\right\} \frac{1}{\omega_{ci}} \quad . \tag{6}$$

In a fusion-oriented context a typical required product of ion cyclotron frequency and confinement time, $\omega_{ci}\tau_c$, could be of order 10^8.

Typical values for $\omega_{ci}\tau_b$ could be 10^2 and for $k_\perp a_i$, 20. Taking $\Delta W_\perp \approx E_i$ as a rough criteria for loss of confinement, and using the above-cited values, an estimate may be derived for the ratio $(e\phi/E_i)$ required in order that the fluctuations should have a negligible effect on confinement (as compared to collisional losses). One finds

$$\frac{|e\phi|}{E_i} < 10^{-4} .$$

Taking E_i = 200 keV, for example, as in the plugs of a tandem mirror, one finds the requirement $|\phi| < 20$ volts. This would appear to be a severe requirement. It is therefore encouraging that in mirror experiments such as 2XIIB /3/ and, more recently, in the tandem mirror experiment TMX-Upgrade /4/ at Livermore, the suppression of high frequency fluctuations has been sufficient to satisfy similarly restrictive requirements on $|\phi|$.

C. Stabilization Techniques for High Frequency Instabilities

The dominant high frequency instability modes in mirror systems, the ones that were suppressed in the aforementioned experiments, are the Drift-Cyclotron Loss Cone (DCLC) mode and the Alfvén Ion Cyclotron (AIC) mode. In brief, the DCLC mode /5/ is an azimuthally-directed electron drift wave driven unstable by ions at the low-energy end of the ion distribution function. The AIC is an Alfvén wave driven unstable (above a critical beta value) by anisotropy ($p_\perp >> p_\parallel$) of the ion distribution function. Although the means for stabilization of these two modes are different, a technique was demonstrated in TMX-U that accomplished both objectives. This technique involves the creation of a hot plasma in the mirror plugs by a combination of angled neutral beam injection and electron cyclotron resonance heating. The use of angled injection ("sloshing ions") increases $p_\parallel$ relative to $p_\perp$, thus drives up the critical beta for the AIC. At the same time it produces a local depression in the ion density midway between the mirrors of the plug cell. Together with the local heating of the electrons caused by ECRH a negative-going potential dip is produced into which "warm" plasma ions may be trapped, thus partially filling in the low energy end of the ion distribution function and thereby stabilizing (or greatly weakening) the DCLC mode.

The AIC instability was first investigated, in an infinite homogeneous plasma, by Rosenbluth /6/ and, independently, by Sagdeev and Shafranov /7/. Later theoretical work, in the context of mirror research, has involved the

inclusion of finite geometry effects, a stabilizing influence. Among papers covering such aspects are papers by Cordey and Hastie /8/, Davidson and Ogden /9/, Rognlien and Watson /10/ and Watson /11/.

Figure (2), taken from the paper by Watson /11/ summarizes the results of detailed calculations including finite geometry effects. Shown plotted are the stability boundaries in $\beta_\perp$, $\beta_\parallel$ space for (1) an infinite plasma; (2) (Labeled "Case I") a mirror-contained plasma with $R_m = 2$ and a ratio of plasma radius to orbit radius $R_p/a_i = 15$ and a ratio of length to orbit radius $L/a = 50$; and (3) (labeled Case II), with $R_m = 2$, $R_p/a_i = 2.7$, and $L/a_i = 15$. Here $\beta_\perp \equiv p_{\perp o}/(p_{\perp o} + B^2/8\pi)$ and $\beta_\parallel \equiv p_{\parallel o}/(p_{\parallel o} + B^2/8\pi)$. The stabilizing effects of finite geometry are evident, especially in Case II.

A convenient, if approximate way to represent the stability results is to give them as a critical product, $\beta_\perp A^2$, of $\beta_\perp$ and an "anisotropy factor" $A \equiv \langle W_\perp\rangle/\langle W_\parallel\rangle$; $A = 1$ for an isotropic plasma. For Case I for example, the critical product $\beta_\perp A^2$ is about 3.0.

The results just cited may be used to illustrate the stabilizing influence of sloshing ion distributions on the AIC mode, as follows:

A useful approximate representation of a mirror-confined ion distribution function is the separated form

$$f(v, \mu_\theta) = h(v)\, g(\mu_\theta) ,$$

with $\mu_\theta = \cos(\theta)$, $\theta = \sin^{-1}(v_\perp/v)$. In this model the anisotropy factor becomes

$$A = \langle W_\perp\rangle/\langle W_\parallel\rangle = \frac{1}{2}\left\{\int_0^1 (1-\mu_\theta^2)\, g(\mu_\theta)\right\} d\mu_\theta \Big/ \left\{\int_0^1 \mu_\theta^2 g(\mu_\theta)\, d\mu_\theta\right\} . \tag{7}$$

As can be seen, an isotropic distribution $[g(\mu_\theta) = g_o = \text{constant},\ 0 \leq \mu_\theta < 1]$ yields the value $A = 1.0$, as required.

As a model of a sloshing ion distribution we will take a gaussian-peaked "injection" distribution in μ_θ, superposed on an approximate "normal mode" angular distribution, representative of a collisionally-randomized trapped ion distribution, taking

$$g(\mu_\theta) = g_{NM}(\mu_\theta)\, g_s(\mu_\theta) \tag{8}$$

$$g_{NM} = [1 - (\mu_\theta^2/\mu_{\theta o}^2)] ,\ 0 \leq \mu_\theta \leq \mu_{\theta o} \tag{9}$$

$$g_s = \exp\left[-(\mu_\theta - \mu_{\theta s})^2/(\Delta\mu_\theta)^2\right] . \tag{10}$$

Here $$\mu_{\theta o}^2 = [1 - 1/R_m],\ \mu_{\theta s}^2 = [1 - 1/R_s] , \tag{11}$$

with R_s being the mirror ratio corresponding to the reflection point of the skewed-angle injection. The parameter $(\Delta\mu_\theta)^2$ is a measure of the dispersion of the distribution function about the mean injection angle, reflecting an estimate of the effect of collisional broadening. A representative value might be $\Delta\mu_\theta = 0.5$. Typical values of R_m and R_s, similar to those used in the TMX-Upgrade experiment, are $R_m = 4.0$ and $R_s = 2.0$.

For the above parameters, the expression for A yields the following results: (1) for 90° injection (i.e., perpendicular to the field lines) $A_{90} = 5.7$, (2) For 45° injection, $A_{45} = 1.7$. The "Case I" corresponding critical $\beta_\perp$ values are $\beta_{\perp c}(90^o) = 0.09$ and $\beta_{\perp c}(45^o) \approx 1.0$, respectively, illustrating the strongly stabilizing effect of angled injection.

D. Tandem Mirror End Losses - The Pastukhov Relation

We conclude the discussion of end losses from mirror systems with a brief review of the theory of electrostatic confinement in a tandem mirror. In a simple tandem mirror system the confining ambipolar potential is derived from the density ratio between the plug plasmas and the central cell through the Boltzmann relationship

$$e\Delta\phi \equiv e(\phi_p - \phi_c) = kT_e \ln(n_p/n_c) . \tag{12}$$

The newer version of the TM, with thermal barriers /12/, has the modified potential relationship

$$e\Delta\phi = kT_{ep}\left(\frac{n_p}{n_b}\right)\left(\frac{T_{ec}}{T_{ep}}\right)^{1/2} - kT_{ec}\left(\frac{n_c}{n_b}\right) . \tag{13}$$

Here n_b is the passing electron density in the thermal barrier, $n_b < n_p$, and T_{ep} is the electron temperature in the plug. Since with the thermal barrier present it is possible to have $T_{ep} >> T_{ec}$, $\Delta\phi$ can be increased over that in a conventional TM, even for $n_p < n_c$.

The effect of the confining potential $\Delta\phi$ on axial confinement can be deduced from an extension of the analytical derivation of Pastukhov /13/, who

treated the case of electrons escaping over the ambipolar potential in a conventional mirror machine. As extended by Cohen, Rensink, Cutler, and Mirin /14/ their expression for the characteristic confinement parameter is approximated by the expression

$$n\tau \approx \frac{\pi^{1/4}}{4} (n\tau_{ii}) \left(\frac{e\Delta\phi}{kT_i}\right) \exp\left(\frac{e\Delta\phi}{kT_i}\right) \ln(4R_m) , \tag{14}$$

where $n\tau_{ii}$ is the Spitzer ion-ion relaxation density-time product given by Eq. (1). Comparing this expression with Eq. (2) giving $n\tau$ for a conventional mirror, it can be seen that the effect of the potential barrier is to multiply the confinement time by the factor

$$E = \left(\frac{e\Delta\phi}{kT_i}\right) \exp\left(\frac{e\Delta\phi}{kT_i}\right) . \tag{15}$$

For $e\Delta\phi$ a few times kT_i this represents a very large increase in $n\tau$. In fact it is not difficult in principle to make this factor large enough that axial losses become essentially negligible, so that radial losses are the controlling factor. In the remainder of this article we will therefore concern ourself entirely with the question of radial transport.

IV. RADIAL CONFINEMENT AND TRANSPORT

In considering the problem of radial particle transport in mirror systems it is important ab initio to recognize a major difference between the situation in mirror systems and that in closed systems such as the tokamak. In the latter systems cross-field particle transport is forced to be ambipolar in nature. That is, in steady state the loss of ions across the field must on average be accompanied by the loss of a charge-equalling number of electrons, usually implying the development of an ambipolar potential sufficient to balance the two rates. Classically this potential can be expected to be negative in a toroidal system, reflecting the slower classical rate of diffusion of electrons across the field as compared to ions. In closed systems it then follows that any process that enhances the cross-field loss rate of one species, say the ions, must at the same time create a situation where the rate of cross-field transport of the other species is similarly enhanced. Not necessarily so in the tandem mirror. In this case, where there are two distinct loss channels, i.e., out the ends or across the field,

classical or other radial ion transport can be balanced solely by axial electron losses, without the necessity for the development of anomalous cross-field transport of electrons in order to satisfy change balance. Since it appears that non-ambipolar cross-field ion transport is expected to be the dominant radial transport process in a TM, in what follows this process will be the main one that is discussed.

A. Neoclassical and Resonant Radial Transport

Radial transport in mirror systems arises from diffusive processes operating on the second adiabatic invariant, J. While particle-particle collisions represent the most obvious such process, geometry-related non-adiabaticities can also play an important role in enhancing radial transport if care is not taken to minimize them. As noted, these nonadiabaticities arise from resonances between the bounce frequency and harmonics of the azimuthal drift frequency generated by departures from axisymmetry. In axisymmetric mirror fields, where azimuthal drift velocities are constant and the drift surfaces are circular in cross section, there are no such resonances, J remains a good invariant and radial transport of nonadiabatic origin does not occur. However, in nonaxisymmetric mirror cells, such as those in present-day tandem mirror systems, quadrupole magnetic wells closely coupled to the central cell distort the axisymmetry of that cell. In such systems azimuthal particle drift velocities are not constant with azimuth, so that bounce-drift harmonic resonances can occur.

1. Analytical Approaches

The analysis of the radial transport effects here being considered starts with consideration of the single-particle orbits of ions in the magnetic field. Whereas in the limit of infinitesimal ion gyroradius the particle guiding centers would remain on well-defined drift surfaces, this is not quite the case when the gyroradius is finite. The guiding center motion, representing the average over the gyrating motion of the local magnetic field, will reflect the presence of the gradients of that field. Thus an ion, starting at the midplane, being reflected at a mirror and returning to the midplane will have its guiding center displaced both azimuthally (drift) and radially inward or outward from its starting position. This radial displacement, call it Δr, is a function of the magnetic moment and energy of the particle and of the initial radius and azimuthal angle (position on the drift surface) of the particle guiding center. The magnitude of Δr can be derived from drift equations, such as those given by Bernstein /15/. As

discussed by Cohen /16/, for example, in a static magnetic field Δr is given by the relation

$$\Delta r(\varepsilon,\mu,r,0) = \frac{c}{eBr}\frac{\partial J}{\partial \theta} , \tag{16}$$

where J is here to be evaluated over the guiding center trajectory between the midplane (where Δr is measured) and the turning point at the mirror, s_t:

$$J = 2\int_0^{s_t} ds\, [2M\,(\varepsilon - \mu B - e\phi)]^{1/2} . \tag{17}$$

In the central cell of a tandem mirror contributions to $\partial J/\partial\theta$ will arise only from the transition regions at the ends (the center region being axisymmetric). If one then makes the reasonable assumption that the value of beta is small in these transition region (where B is high), then J may be evaluated from the vacuum field configuration, by expanding the integrand in Eq. (17) in powers of r_v^2, the radius of a given field line in the vacuum field. To leading order in r_v, Cohen finds

$$\Delta r \approx \frac{a_i r_v}{L_{tr}} K(v_\perp^2/v^2)\, \sin\,(2\theta) , \tag{18}$$

where L_{tr} is the length of the transition region and a_i is the (vacuum field) ion orbit radius. Here $K(v_\perp^2/v^2)$ is a function of the pitch angle, $\theta = \sin^{-1}(v_\perp/v)$, that must be evaluated for a given field configuration by, for example, particle orbit-following using a computer codes, such as the Livermore TIBRO code. As an example, for the Livermore TMX tandem mirror devices Cohen et al. /17/ found an approximate representation

$$K = 4.57(v_\perp/v)^2 - 4.51(v_\perp/v)^4 . \tag{19}$$

Looking somewhat deeper into the origin of Δr it can be discerned that it arises from the "geodesic" curvature of the field lines. That is, it arises from the component of field line curvature representing bending of the field lines in a direction tangent to the flux surface, as contrasted to the "normal" component of curvature. From the drift picture it is easy to see that while the "normal" curvature leads to drifts lying in the flux surface, i.e., in the θ direction, geodesic curvature will lead to drifts

perpendicular to that surface. Axisymmetric fields, having no geodesic curvature, have only θ drifts so that $\Delta r = 0$ identically. From this picture the origin of the "sin (2θ)" term in the expression for Δr also becomes evident. As illustrated in Fig. (1), the geodesic curvature is zero in the symmetry planes defined by $\theta = 0, \pi/2, \pi$, and $3\pi/2$.

As might be suspected from the θ-dependence of Δr, the radial particle transport associated with Δr depends very much on the magnitude of $\Delta\theta$, the azimuthal drift per bounce. There are here two regimes with very different characteristics:

(1) $\Delta\theta \ll 1$. In this regime the transport is relatively slow, resembling the "neoclassical" regimes of the tokamak.

(2) $\Delta\theta$ of order unity. Here so-called "resonant" transport can occur with rapid radial losses in certain circumstances.

Within which regime a given system will be found will depend both on geometric and scale factors, i.e., on the field configuration and size, and on plasma parameters such as beta and ambipolar potential. Finite beta in the central cell of a tandem mirror will introduce azimuthal magnetic drifts; the ambipolar potential, positive in the normal tandem mirror, will introduce azimuthal electric drifts. Being oppositely-directed for ions, beta-induced and ambipolar positive-induced drifts may cancel at particular radii, giving rise to special effects such as "bananas".

The various transport phenomena associated with the regimes just discussed can all be analyzed in terms of the combined effects of: (1), the θ and r-directed drifts and (2), the role of collisions (or other processes) in diffusing particles within the phase space of these regimes.

Following the techniques put forth by Chirikov /18/, as applied by Ryutov and Stupakov /19/ and by Cohen /20/, the analysis of radial transport may be discussed in terms of "mapping equations". These equations describe the successive incremental changes in the r and θ coordinates of particles as they pass through the midplane during the course of their longitudinal bouncing motion between the mirrors. With the index n counting the number of reflections from the mirrors, and with $\Delta r = r_{n+1} - r_n$ and $\Delta\theta = \theta_{n+1} - \theta_n$, the equations are*

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*These equations have been written without the addition of a usually-employed higher-order term arising from quadrupolar corrections to the drift surfaces. Nevertheless, it can be shown that these equations are able to generate banana orbits through second-order effects that are implicit to their form.

$$r_{n+1} = r_n + (-1)^n a_1 \sin(2\theta_n) \tag{20}$$

$$\theta_{n+1} = \theta_n + (-1)^n b_1 \cos(2\theta_n) + b_d \ . \tag{21}$$

In order that the mapping be "area preserving," i.e., physical, the terms $a_1(r_{n+1})$ and $b_1(r_{n+1})$ are necessarily related through

$$b_1 = \frac{1}{2} \frac{\partial a_1}{\partial r_{n+1}} \ . \tag{22}$$

In a quadrupole tandem mirror field, as noted earlier, we have

$$a_1 = \frac{a_i r_{n+1}}{L_{tr}} K(v_\perp^2/v^2) \ . \tag{23}$$

The important terms $(-1)^n$ arise from the symmetry properties of quadrupolar tandem mirror fields, in which the "fishtails" at each end are turned 90° in azimuth with respect to each other (see Fig. 1). If they are so rotated, successive dispacements Δr will cancel to first order, as can be seen from the equations. If they were not so rotated, this cancellation would not occur, leading to rapid radial drifts in the case of small $\Delta\theta$.

The term $b_d(r_{n+1})$ represents azimuthal drifts, such as those associated with finite beta effects or with ambipolar electric fields. The regime, i.e., "neoclassical" or "resonant," in which the transport occurs depends on the magnitude of b_d, with $b_d << 1$ being associated with the former. For situations where b_d is larger, so that $b_d = k\pi/2$, k = 1, 3, 5 ---, for a non-vanishing fraction of the particles, then, as can be seen from the equations, the 90° rotation of the fans will be cancelled out at each successive reflection so that the Δr's will be additive, leading to a "resonant" radial walking out of that class of particles, and thus to greatly enhanced radial transport for particles in the vicinity in phase space as collisions move them into such resonant zones.

Though it is not likely to be encountered in realistic situations, a still more virulent regime, the "stochastic" regime, can be encounterd if the magnitude and radial variation of the azimuthal drift is such that higher order resonances (k = 3,5, etc.) can occur at radii adjacent to the first order resonance. In such cases of overlapping resonances, rapid transport can occur even in the absence of collisions, solely induced by the bounce-drift

resonances. As it turns out, present-day tandem mirror devices usually lie somewhere between the neoclassical and the resonant regimes and do not encounter stochastic radial transport.

Insight into the processes involved can be gained by examining solutions to the mapping equations in various parameter ranges. These equations have been programmed for iterative solution on a computer.* Examples of these solutions will be presented here, as case studies, to illustrate the various regimes involved. (The magnitudes of the radial transport associated with these regimes will be discussed later.)

Case I "Classical": $b_d = b_{do}$ = constant, $0 > b_{do} << 1$ (small, constant, positive drift).

This case might be roughly representative of the situation in a short, quadrupole magnetic well (such as the plug region in a tandem mirror), with a moderate level radial ambipolar electric field providing the drift. Figure (3) shows a polar plot of 200 successive crossings of a particle for which $a_1 = 0.1$, $b_{do} = 0.1$. These parameters are roughly representative of those in the plugs of a tandem mirror experiment such as TMX. They correspond to a 10 keV deuteron, trapped in a 200-cm long, 1-T mirror field, with $L_{tr} = 100$ cm, and drifting azimuthally at a radius of 10 cm in a radial electric field of 140 volts/cm. As can be seen, the midplane crossings lie on nearly circular curves whose fractional deviations from circularity are small, of order 1 cm, (as compared to the ion orbit diameter of order 4 cm) Radial transport in such a situation would be expected to be minimal. In Fig. (4) the same data for r are plotted against the number of crossings through the midplane.

Case II "Neoclassical": $|b_d| << 1$; b_d reverses sign at $r = r_o$.
This situation corresponds to one in which the drift term, though relatively small, results from a competition between beta-induced and ambipolar electric-induced drifts, such that the drift vanishes at a particular radius. In this circumstance, banana orbits are formed, bounded radially by drifting orbits that encircle the axis. Depending on conditions, the banana orbits may have a substantial radial extent, so that the associated radial transport will be enhanced over the classical value.

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*The r equation (Eq. 20) was normalized to the unit circle in solving the equations, i.e., in what follows the coefficient a_1 corresponds to the fractional displacement in r relative to 1 cm caused by the drift. The r values shown are correspondingly normalized relative to 1 cm.

Figure (5) shows banana orbits generated by 200 crossings for a case where $a_1 = 0.1$ (as before) and $b_d = 0.1\ (r-10)$, i.e., the drifts vanish at $r = 10$ cm. This example would again be representative of 10 keV deuterons in a 1 T field with $L_{tr} = 100$ cm, but now subject to both a beta-induced drift and azimuthally-directed electric drift producing drift cancellation at $r = 10$ cm. Both banana orbits and adjacent circling drifts as calculated for ions at smaller and larger radii are shown. Comparing the radial excursions for this case with those for Case I, it can be seen that the step size for radial diffusion is substantially larger.

<u>Case III</u> "Resonant" diffusion: $b_d = k\ \pi/2$, $k = 1, 3, 5$ at some radius. This situation corresponds to that which can arise in the central cell of a tandem mirror system when electric and/or beta-induced drifts result in azimuthal drifts of odd multiples of $\pi/2$ per bounce. Here the effects of resonance depend on the way in which the drift varies with radius. If the angular velocity of the drift is constant with radius, then at exact resonance, the radial increments for the particles will be directly accumulative, so that the excursions will be very large, and the associated radial transport would be extremely rapid. In a more realistic situation, for a given particle group, exact resonance will only occur at some critical radius, resulting now in "resonant" bananas. These resemble qualitatively the neoclassical bananas, but are generally much larger in radial width, implying correspondingly increased radial transport. Figure (6) illustrates this behavior, for the same value, $a_1 = 0.1$, as for the previous cases, but now with $b_d = \pi/2(r/10)$, i.e., with a drift-induced resonance at $r = 10$ cm. As can be seen from the figure, for the initial conditions chosen, the radial widths of the bananas are large compared to the neoclassical ones. Figure (7) is a plot of the radial excursions vs. the number of crossings.

As the three case studies just presented illustrate, for the same value of fractional radial drift coefficient per bounce ($a_1 = 0.1$ in the examples) the resultant radial excursions, which determine the step size in the radial transport problem, vary from smaller than an ion gyroradius to values comparable to the plasma radius itself, depending on the magnitude and radial dependence of the drift term, b_d. In what follows, we will briefly review the results of the analysis of the radial transport that is predicted to arise in the various regimes discussed, identifying problem areas. In a later

section we will address possible ways to avoid or prevent the occurrence of the high rates of transport that are predicted to be associated with the resonant regime.

2. Review of Approximate Analytical Results: Neoclassical Diffusion

The method of obtaining approximate analytical results for the radial transport in the regimes discussed parallels that used in estimated transport in tokamaks. A radial diffusion coefficient is defined in terms of a radial step size, call it δr, an "effective" collision frequency, ν_{eff}, and a fraction, f, of the particle population that participates. Then

$$D = (\delta r)^2 \, f \, \nu_{eff} \qquad cm^2 \, sec^{-1} \, . \tag{24}$$

Except for the "classical" regime (for which the step sizes are small, as is the resultant transport), the step size is given by or related to the banana width in the neoclassical and resonant banana regimes. The effective collision frequency is the reciprocal of the Spitzer ion-ion relaxation time enhanced by the factor $(\Delta W_\perp/W_\perp)^{-2} = (\Delta\mu/\mu)^{-2}$ to take into account the fact that at a given radius the fractional range in μ over which banana orbits exist (adjacent to passing orbits) is $(\Delta\mu/\mu)$. Thus to effectuate a step requires only that μ should diffuse by an amount $\Delta\mu$ from this special value. Finally, and for similar reasons, the fraction of the particles participating in these banana-to-passing transitions is given by the quantity $(\Delta\mu/\mu)$. It follows that D can be estimated if estimates for δr and $\Delta\mu/\mu$ can be determined from the details of the orbits. Such estimates were obtained by Ryutov and Stupakov /21/ and form the basis of their early discussion of radial transport. From the definition of J and from the vacuum field configuration they find for a tandem mirror central cell with length L_c, capped by nonaxisymmetric end mirrors with length L_{tr} the following: Define the dimensionless parameter

$$\alpha = \frac{r_c^4}{L_c L_{tr}^3} \, , \tag{25}$$

with r_c being the plasma radius. Then for the banana width, δr, one finds

$$\frac{\delta r}{r} \approx \alpha^{1/2} \, (r/r_c) \, . \tag{26}$$

In terms of α the fractional spread in μ of a banana is $(\Delta\mu/\mu) \approx \alpha^{1/2}$, so that the lifetime on a banana orbit is

$$\tau_{eff} = \nu_{eff}^{-1} = \alpha\nu_i^{-1} \ . \tag{27}$$

The lifetime to drift around a banana orbit is found to be $\tau_{ban} \approx \alpha^{-1/2}\,\omega_{dr}^{-1}$, where ω_{dr} is the azimuthal drift frequency. Depending on a comparison between this time and τ_{eff}, the regime will be describable as "banana", "plateau" or "collisional" diffusion. When $\nu_{eff} << \tau_{ban}$, i.e., when $\nu_i\tau_{dr} << \alpha^{3/2}$, the step size is the full banana width so that

$$D_{nb} = (\delta r)^2\, f\, \nu_{eff} \approx (\alpha^{1/2} r_c)^2 (\alpha^{1/2})(\nu_i/\alpha) = \alpha^{1/2}\, r_c \nu_i \ . \tag{28}$$

In the intermediate (plateau) regime when $\alpha^{3/2} < \nu_i\tau_{dr} < 1$, particles in a wider range of μ-space can scatter into banana orbits during one τ_{ban}, as determined by the relationship

$$\left(\frac{\Delta\mu}{\mu}\right)^2 \frac{1}{\nu_i} \approx \tau_{ban} \approx \tau_{dr} \left(\frac{\mu}{\Delta\mu}\right) \ , \tag{29}$$

so that

$$\left(\frac{\Delta\mu}{\mu}\right)^3 \approx \nu_i\tau_{dr} \ . \tag{30}$$

Both present experiments and fusion tandem mirror fusion systems lie in the plateau regime. For this regime, in which frequent collisions occur during one banana orbit time, it is appropriate to use an alternative definition of D in obtaining an estimate. It is

$$D = \langle \text{radial velocity} \rangle^2 \cdot (\text{correlation time}) \cdot f \ . \tag{31}$$

Here the appropriate radial velocity may be estimated by dividing the radial width of a banana, $\delta r \approx \alpha^{1/2}\, r_c$, by the period of a banana orbit, $\tau_{ban} \approx \alpha^{-1/2}\omega_{do}^{-1} = \alpha^{-1/2}\tau_{dr}$. So that $\langle v \rangle \approx \alpha\, r_c\, \omega_{do}$. The correlation time is $\nu_{eff}^{-1} \approx \alpha\nu_i^{-1} = (\Delta\mu/\mu)^2\, \nu_i^{-1}$, and the fraction of particles participating is $\Delta\mu/\mu$, giving for D

$$D_n \approx \alpha^2 (\omega_{do}^2 \nu_i^{-1}) \left(\frac{\Delta\mu}{\mu}\right)^3 r_c^2 \ . \tag{32}$$

From Eq. (30) we have therefore in the plateau regime (independent of ν_i)

$$D_{np} \approx \alpha^2 r_c^2 \tau_{dr}^{-1} \ . \tag{33}$$

The corresponding particle lifetime is therefore

$$\tau_{np} \approx D_{np}^{-1} r_c^2 = \alpha^{-2} \tau_{dr} \ . \tag{34}$$

Inserting values of α and τ_{dr} appropriate to TMX and for a fusion TM, one finds

$$\tau_{np} \approx \begin{cases} 10^6 \ \nu_i^{-1} & \text{TMX} \\ 10^7 \ \nu_i^{-1} & \text{Fusion TM} \end{cases}$$

These times are clearly much longer than the axial lifetimes in either case (of order 100 ν_i^{-1} for both cases). One concludes therefore that neoclassical diffusion times do not represent an important constraint on the lifetime of particles in TM systems. Of much greater relevance are the resonant transport processes to be discussed in the next section.

3. Approximate Analytical Treatment of Resonant Diffusion

The analysis of neoclassical transport just discussed was based on the assumption that axial bounce frequencies are sufficiently high relative to azimuthal drift frequencies that J-breaking resonances between the two were not important and therefore that J could be considered a good invariant. TM systems can in principal be designed to satisfy this requirement, even in nonaxisymmetric geometries. However, under commonly encountered TM machine and plasma parameters the drift frequency requirement may not be satisfied. This situation may be illustrated by estimating the magnetic and electric drifts that may occur in the central cell of a TM.

For the magnetic drifts we have

$$\omega_d = v_d/r \approx [v_\perp a_i (\frac{1}{B}\frac{dB}{dr})]/r \tag{35}$$

$$\omega_b \approx v_{\parallel}/L_c \ . \tag{36}$$

We may estimate the beta-induced magnetic drifts from the approximate relationship (appropriate for a parabolic distribution of the radial pressure):

$$\frac{1}{B}\frac{dB}{dr} \approx \beta_o \, r/r_c^2 \, . \tag{37}$$

It follows that

$$\left.\frac{\omega_d}{\omega_b}\right|_{\text{magnetic}} \approx \beta_o \left(\frac{a_i L_c}{r_c^2}\right)\left(\frac{v_\perp}{v_\parallel}\right) \, . \tag{38}$$

Similarly, with a parabolic potential profile having the magnitude on axis (relative to the mean ion energy) of

$$e\phi_o = \eta_o W_i \, , \tag{39}$$

there results for the electric drift case

$$\left.\frac{\omega_d}{\omega_b}\right|_{\text{elec.}} \approx \eta_o \left(\frac{a_i L_c}{r_c^2}\right)\left(\frac{v_\perp}{v_\parallel}\right) . \tag{40}$$

The common factor $(a_i L_c/r_c^2)$ is roughly unity in present TM systems. Thus, disregarding the anisotropy factor $(v_\perp/v_\parallel)$ it can be seen that either finite beta values or radial potential variations of the order of the ion energy (as may be encountered in a TM) can result in J-breaking resonances.

To estimate the rates of resonant transport we will return to the mapping equations previously cited and derive approximate analytical results, drawing on a discussion of the problem by D. Baldwin [unpublished].

As noted earlier, when for a given $r = r_k$ the drift, $\Delta\theta(r)$, is large enough to satisfy the resonance relationship $\Delta\theta(r_k) = k\pi/2$, $k = 1,3,5$---, then for that class of particles the Δr values will add constructively. As was shown in the numerical solutions of the mapping equations (Case IV), banana orbits will be generated as particles pass through these "resonant surfaces". We can estimate the parameters of these bananas by passing to the continuum limit of the mapping equations (i.e., $\tau_b \ll \tau_{ban}$) and approximating them as differential equations in the vicinity of a resonant region, finding:

$$\dot{r} = (-1)^{\frac{k+1}{2}} \frac{2|\Delta r|}{\tau_b} \sin 2\theta \tag{41}$$

$$\dot{\theta} = \frac{r - r_k}{\tau_b} \theta'(r_k) \ , \tag{42}$$

where $\theta' \equiv [\partial\theta/\partial r]_{\varepsilon,\mu}$ and $\tau_b = L_c/v_{\parallel}$. As noted previously, $|\Delta r| = a_1$ may be estimated as being approximately equal to $(a_i r_c/L_{tr})$. Linearizing the $\dot{r}$ equation, differentiating the $\dot{\theta}$ equation and substituting for $\dot{r}$ results in a "pendulum" equation for θ from which the banana period and width may be estimated. We have:

$$\ddot{\theta} \approx (-1)^{\frac{k+1}{2}} \left[\frac{2\Delta r}{\tau_b^2} \theta'\right] \theta \ , \tag{43}$$

from which one finds for the period of a resonant banana

$$\tau_{rb} \approx \tau_b \left|\frac{1}{\Delta r \theta'}\right|^{1/2} \ . \tag{44}$$

From this it follows that

$$(\Delta r)_{res} \approx \int_0^{\tau_{rb}} \dot{r} dt \approx \left|\frac{\Delta r}{\theta'}\right|^{1/2} \ . \tag{45}$$

As in the case of neoclassical diffusion, different degrees of collisionality will lead to different types of diffusion, i.e., "resonant banana," "plateau," and "collisional".

Again, it turns out that TM systems are most likely to operate in the plateau regime, for which estimates of D_{rp} are derived below. Thus, when $\nu_{eff}\tau_{res} > 1$ the following considerations apply:

For resonant particles the radial displacements will accumulate so that

$$(\delta r)_{res} \approx \dot{r}\ \tau_{eff} \approx \left(\frac{\Delta r}{\tau_b}\right)\left(\frac{\Delta\mu}{\mu}\right)^2 \tau_i \ . \tag{46}$$

For banana particles the mapping differential equations may be written in a form relating $(\delta r)_{ban}$ to $\Delta\mu$, i.e.,

$$\dot{r} \approx \frac{\Delta r}{\tau_b} \sin 2\theta \tag{47}$$

$$\dot{\theta} \approx \frac{1}{\tau_b}\left[\frac{\partial(\Delta\theta)}{\partial\mu}\right] \Delta\mu \tag{48}$$

i.e.,

$$\theta \approx \frac{1}{\tau_b}\left[\frac{\partial(\Delta\theta)}{\partial\mu}\right] (\Delta\mu)\ t \tag{49}$$

substituting in the r equation and integrating there results

$$(r - r_o) \approx \frac{\Delta r}{2\left[\frac{\partial(\Delta\theta)}{\partial\mu}\ \Delta\mu\right]} \cos 2\theta\ . \tag{50}$$

From this we find

$$(\delta r)_{ban} \approx |r - r_o| = \frac{\Delta r}{2\left[\frac{\partial(\Delta\theta)}{\partial\mu}\ \Delta\mu\right]}\ . \tag{51}$$

Now equating $(\delta r)_{ban}$ and $(\delta b)_{res}$ determines the range of $\Delta\mu$, i.e.,

$$\left(\frac{\Delta r}{\tau_b}\right)\left(\frac{\Delta\mu}{\mu}\right)^2 \tau_i = \frac{\Delta r}{2\left[\frac{\partial(\Delta\theta)}{\partial\mu}\ \Delta\mu\right]} \tag{52}$$

$$(\Delta\mu)^3 \approx \frac{\mu^2}{2\left[\frac{\partial(\Delta\theta)}{\partial\mu}\right]} \left(\frac{\tau_b}{\tau_i}\right)\ . \tag{53}$$

The diffusion may now be estimated in this (plateau) regime:

$$D_{rp} \approx (\delta r)^2\ f\ \nu_{eff} \approx (\delta r)^2 \left(\frac{\mu}{\Delta\mu}\right) \nu_i\ . \tag{54}$$

Substituting for (δr) and $\Delta\mu$ there results

$$D_{rp} \approx \frac{1}{\left[2\mu\ \frac{\partial(\Delta\theta)}{\partial\mu}\right]}\ \frac{(\Delta r)^2}{\tau_b}\ . \tag{55}$$

The term $2\mu[\partial(\Delta\theta)/\partial\mu]$ is of order unity in the fast drift case, so that the estimate for the resonant plateau diffusion coefficient becomes

$$D_{rp} \approx \frac{(\Delta r)^2}{\tau_b} . \tag{56}$$

The resultant radial transport, arising from step sizes of order Δr at a bounce frequency rate, can be orders of magnitude more rapid than neoclassical rates. Whether these higher rates will in fact compete with axial losses depends on the details of machine design and on operating plasma parameters. We will later discuss measures that may be employed to ameliorate this kind of resonant transport.

To see the order of magnitude of the radial transport times expected in resonant plateau diffusion we insert the definition of Δr in Eq. (56), finding for the lifetime against radial loss

$$\tau_{rp} \approx \left(\frac{L_{tr}}{a_i}\right)^2 \tau_b . \tag{57}$$

In TMX, where $L_{tr} \approx 10^2$ cm, $a_i \approx 1$ cm and $\tau_b \approx 10^{-4}$ sec., this expression leads to a predicted lifetime of about 30 ms, comparable to the axial lifetimes that were characteristic of this device. In a fusion TM, with higher B and larger dimensions the estimated lifetimes are several seconds, however again being short enough to be of some concern.

More precise evaluations of resonant transport rates obtained by Cohen /16/, Cohen and Rowlands /22/, and Mirin et al. /23/ provide a more rigorous basis for the evaluation of radial transport but do not alter the general conclusions obtained from the approximate treatment outlined above.

The preceding discussion has been concerned with situations in which radial transport is associated with banana orbits, centered about a resonant surface, that have been implicitly assumed not to overlap with those associated with higher k values (more rapid drifts). This situation seems to be the case in TM systems now envisaged. However, in the situation where drift frequencies are so high and vary so rapidly with radius that bananas overlap, so-called "stochastic" diffusion (as described by Chirikov /17/ for example) can occur with even higher rates of radial transport than those given by the expression derived here. Needless to say the design of a tandem mirror with quadrupolar fields should avoid stochastic regimes.

B. Other Sources of Radial Transport: Azimuthal Electric Fields

The discussion of neoclassical and resonant radial transport was concerned with the coupling between azimuthal drifts of magnetic and/or electric origin and the azimuthal asymmetry of quadrupolar fields. There are also other circumstances which can cause radial transport, at enhanced rates, if care is not taken to avoid the cause. One such circumstance is the presence of azimuthally-directed electric fields. In a tandem mirror system such electrostatic fields might arise, for example, from azimuthal variations in the intensity of ECRH heating power used in creating a thermal barrier region in the plugs.

The radial diffusion associated with particles sampling such azimuthal fields can be estimated from simple considerations. If it assumed that the drifts in the central cell are such as to remove the correlation between encounters with azimuthal electric field components at the ends then the diffusion coefficient can be estimated from the step size, call it δr, and the repetition period of the step, here equal to τ_b, the bounce time, yielding

$$D_E \approx \frac{(\delta r)^2}{\tau_b} \quad . \tag{58}$$

The magnitude of (δr) may be estimated from the time that the particle spends in the azimuthal electric field, assumed to extend over the length $L_E << L_c$. Thus:

$$\delta r \approx \left(\frac{L_E}{L_c}\right) \tau_b \left(\frac{E_\theta}{B}\right) c \quad ,$$

from which it follows that

$$D_E \approx \left(\frac{L_E}{L_c}\right)^2 \left(\frac{E_\theta}{B}\right)^2 c^2 \tau_b \ . \tag{59}$$

The associated particle lifetime against radial transport is therefore

$$\tau_r \approx \tau_b \left[r_p^2 \left(\frac{L_c}{L_E}\right)^2 \frac{1}{c^2 \tau_b^2} \left(\frac{B}{E_\theta}\right)^2 \right] \tag{60}$$

Since $\tau_b \approx L_c/v_{th}$, this equation may be written in the form

$$\tau_r \approx \tau_b \left[\left(\frac{r_p}{L_E}\right)^2 \left(\frac{v_{th}}{c}\right)^2 \left(\frac{B}{E_\theta}\right)\right]. \tag{61}$$

From this expression a limit on the strength of the azimuthal electric field components (if confinement is not to be compromised) may be deduced. For example, in a TM fusion power system one might have $\tau_r > 1$ second required, with $\tau_b \approx 10^{-4}$ sec. If $r_p \approx L_E$ and $v_{th} = 2 \times 10^8$ cm sec^{-1} (40 keV deuterons) then for B = 3T one finds the requirement

$$E_\theta < 600 \text{ V/cm}.$$

For TMX-U a similar calculation yields

$$E_\theta < 30 \text{ V/cm}.$$

In both cases these electric fields are small compared to the fields from the ambipolar potentials in the plugs, illustrating the care that must be taken to minimize azimuthal electric field components.

Note that the diffusion, if it obeys the relationship given, is non-ambipolar in nature (as is resonant diffusion). Because of the (v_{th}/c) dependence of τ_E, the radial lifetime of electrons if of order $(M/m)^{1/2}$ greater than that of the ions.

C. Some Means for Minimizing Radial Transport

The estimates that have been given show that under not unlikely circumstances radial transport can limit particle lifetimes in a tandem mirror system to values less than that set by axial plugging losses. There are, however, means that can be taken in the design of such systems to reduce radial transport, so as to alleviate this problem. Some of these means, apart from simple scaling considerations, will be discussed in what follows.

1. Choke Coils

Since the diffusion caused by nonaxisymmetric fields depends on the fraction of the time that particles are exposed to these fields one straightforward means for reducing radial transport is to introduce an

axisymmetric mirror or "choke coil" at the ends of the central cell. If the mirror ratio associated with this coil is R_c, then only a fraction approximately equal to R_c^{-1} of the particles at any given instant will be able to penetrate through the choke coil and be exposed to azimuthally-varying fields in the transition and plug coils. Factors of 5 to 10 reductions in radial transport are feasible with the case of high fields. Similar or possibly greater improvement should be possible in the "axicell" design of, for example, the TARA experiment at M.I.T., where the inboard plugs are axially symmetric.

2. Potential Control

Since radial electric fields in the central cell of a tandem mirror, if sufficiently high, can induce electric drifts that result in resonant transport, their control can be an important step in the reducing transport. One suggested method of accomplishing this control is the use of special electrodes at the outer ends (beyond the mirrors) in a tandem mirror. Already tested in a preliminary way in TMX-Upgrade /24/, the idea takes advantage of (1) the circumstance that electron cross-field transport in mirror systems is characteristically less rapid than ion transport and, (2) the overall loss rate must be equal for electrons and ions, even if not through the same channel (i.e., electron loss can be possibly axial while ion loss is mainly radial). The approach is as follows:

Two or more concentric metallic plates, mounted on insulators, are located at each end of the machine. These plates are elliptical in outline, matching to the flux surfaces beyond the fanning field lines outside the quadrupolar mirrors. In operation these plates may be electrically isolated or grounded. When isolated they perforce adjust to potentials such that the ion and electron currents to them are in balance. If the electrons have small transport across the field lines this potential distribution must therefore be one that inhibits the ion transport, for example, it would tend to modify the azimuthal drift in such a way as to avoid resonant transport regimes. With the use of several concentric plates the radial potential distribution should approach an optimal distribution, relative to reducing ion radial transport. Preliminary tests (with two plates) in TMX-U show a measurable decrease in radial ion transport. Figure (8) is a photograph of the plates used in these tests.

3. High-order Multipole Fields

Another design approach that is being examined theoretically is the substitution of higher order multipole fields (octupole in the studies) for the quadrupolar fields usually used in the plugs to achieve MHD stabilization. These studies indicate that the Δr and $\Delta\theta$ values associated with multipole fields tend to be much smaller (as expected) than those for a quadrupole, except at large radii. Though smaller drifts are required to reach the resonant regime, the resultant bananas appear to be much smaller, leading to lowered rates of transport, even in the stochastic regime.

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REFERENCES

/1/ KILLEEN, J., MIRIN, A., and RENSINK, M., Meth. In Computational Physics 16 (1976) 389.

/2/ BERK, H., and STEWART, J., Phys. Fluids 20 (1977) 1080.

/3/ LOGAN, B. G., et al., Phys. Rev. Lett. 37 (1976) 1468.

/4/ SIMONEN, T., et al., Phys. Rev. Lett. 50 (1983) 1668.

/5/ POST, R., and ROSENBLUTH, M., Phys. Fluids 9 (1966) 730.

/6/ ROSENBLUTH, M., Internatl. Summer Course in Plasma Physics, Risö Rpt. 18 (1960) 189.

/7/ SAGDEEV, R., and SHAFRANOV, V., Zh. Eksp. Teor. Fiz., 39 (1960) 181.

/8/ CORDEY, J., and HASTIE, R., Phys. Fluids 15 (1972) 2291.

/9/ DAVIDSON, R., and OGDEN, J., Phys. Fluids 18 (1975) 1045.

/10/ ROGNLIEN, T., and WATSON, D., Phys. Fluids 22 (1979) 1958.

/11/ WATSON, D., Phys. Fluids 23 (1980) 2485.

/12/ BALDWIN, D., and LOGAN, B. G., Phys. Rev. Lett. 43 (1979) 1318.

/13/ PASTUKHOV, V., Nucl. Fusion 14 (1974) 3.

/14/ COHEN, R., et al., Nucl. Fusion 18 (1978) 1229.

/15/ BERNSTEIN, I., Advances in Plasma Physics 4 (1971) 311.

/16/ COHEN, R., Nucl. Fusion 19 (1979) 1579.

/17/ CHIRIKOV, B., Sov. J. Plasma Phys. 4 (1978) 289.

/18/ CHIRIKOV, B., Sov. J. Plasma Phys. 5 (1979) 492.

/19/ RYUTOV, D., and STUPAKOV, G., Sov. Phys. Dokl. 23 (1978) 412.

/20/ COHEN, R., Comments Plasma Phys. Cont. Fusion 4 (1979) 157.

/21/ RYUTOV, D., and STUPAKOV, G., Sov. J. Plasma Phys. 4 (1978) 278.

/22/ COHEN, R., and ROWLANDS, G., Phys. Fluids 24 (1981) 2295.

/23/ MIRIN, A., et al., Nucl. Fusion 23 (1983) 703.

/24/ HOOPER, E., et al. (to be published).

FIGURE CAPTIONS

Fig. 1. Schematic of simple tandem mirror, showing nonaxisymmetric flux surfaces and profiles of magnetic field, plasma density and potential.

Fig. 2. Stability boundaries, in $\beta_\perp$, $\beta_\parallel$, coordinates, for AIC instability, showing stabilizing effects of finite-geometry corrections. See text for explanation of legends.

Fig. 3. Plot of solution to mapping equations for case of small constant drift frequency. 200 crossings, $a_1 = 0.1$, $b_d = 0.1$.

Fig. 4. Plot of radial positions vs number of crossings for example in Fig. 3.

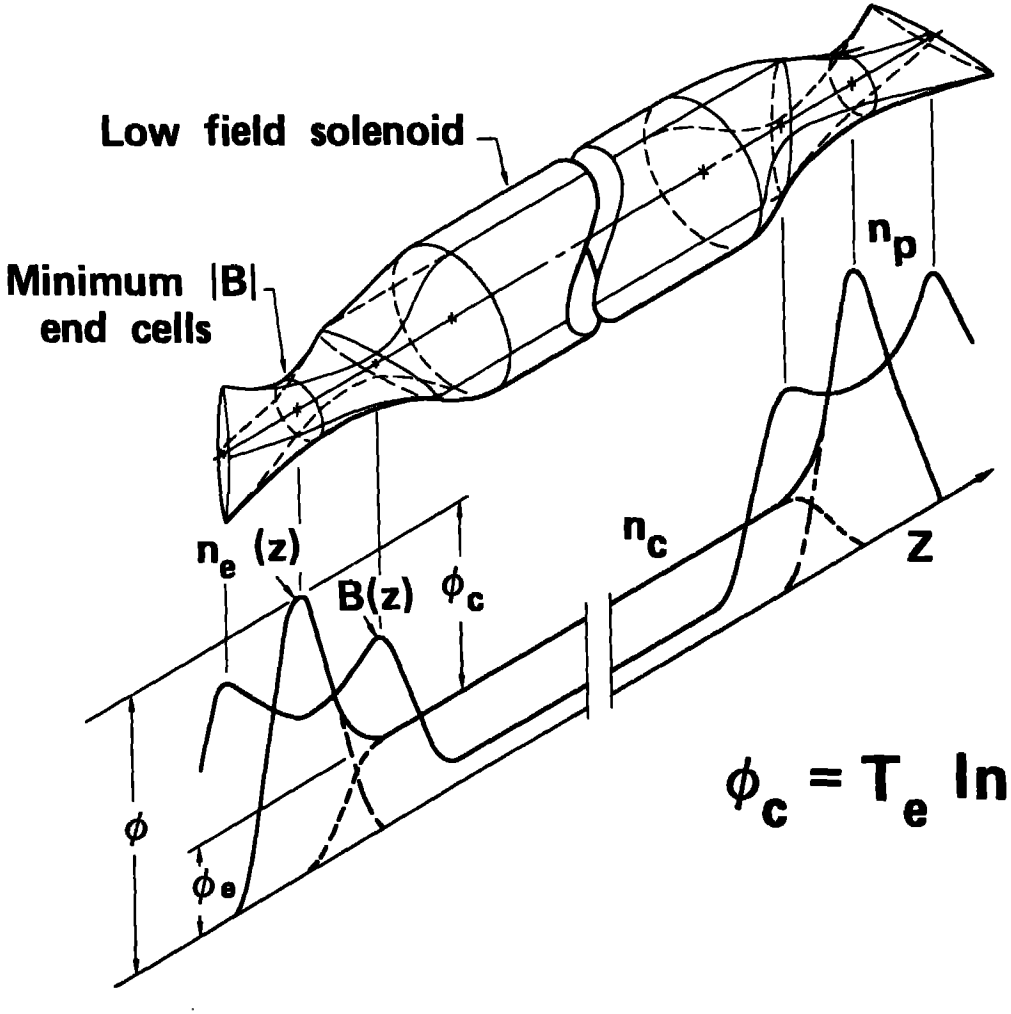
Fig. 5. Plot of solution to mapping equations for case of drift frequency reversal at critical radius, 200 crossings, $a_1 = 0.1$. Banana and passing orbits shown.

Fig. 6. Plot of solution to mapping equations for resonant banana case, showing bananas and passing orbits. 200 crossings, $a_1 = 0.1$, $b_d = \pi/2\ (r/10)$ (i.e., $\pi/2$ at normalized radius $r = 1.0$).

Fig. 7. Plot of radial positions vs number of crossings for example of Fig. 6.

Fig. 8. Photograph of elliptical end plates used in first tests of potential control in TMX-U.

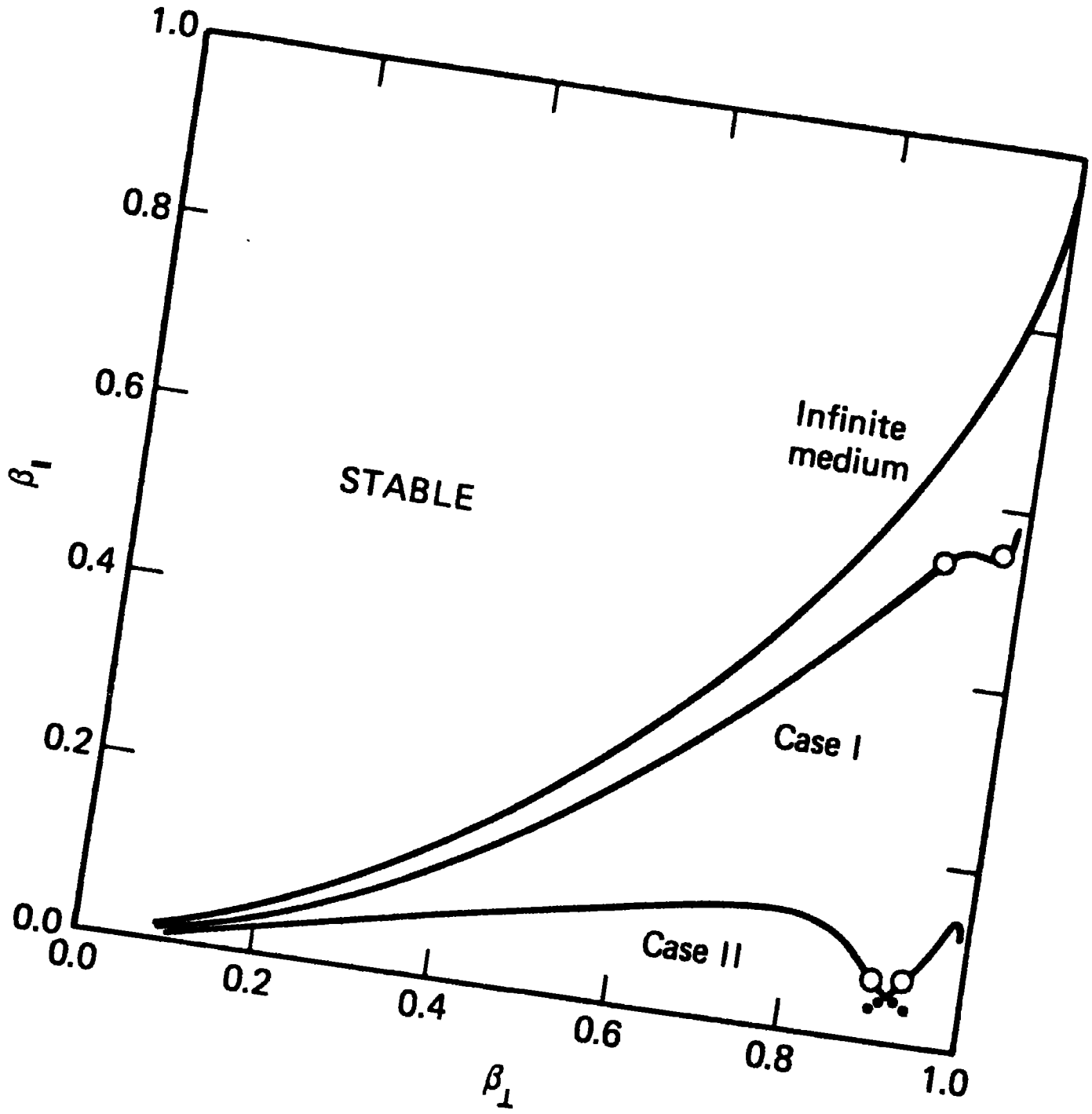
TANDEM MIRRORS WITH AMBIPOLAR BARRIERS AT THE ENDS



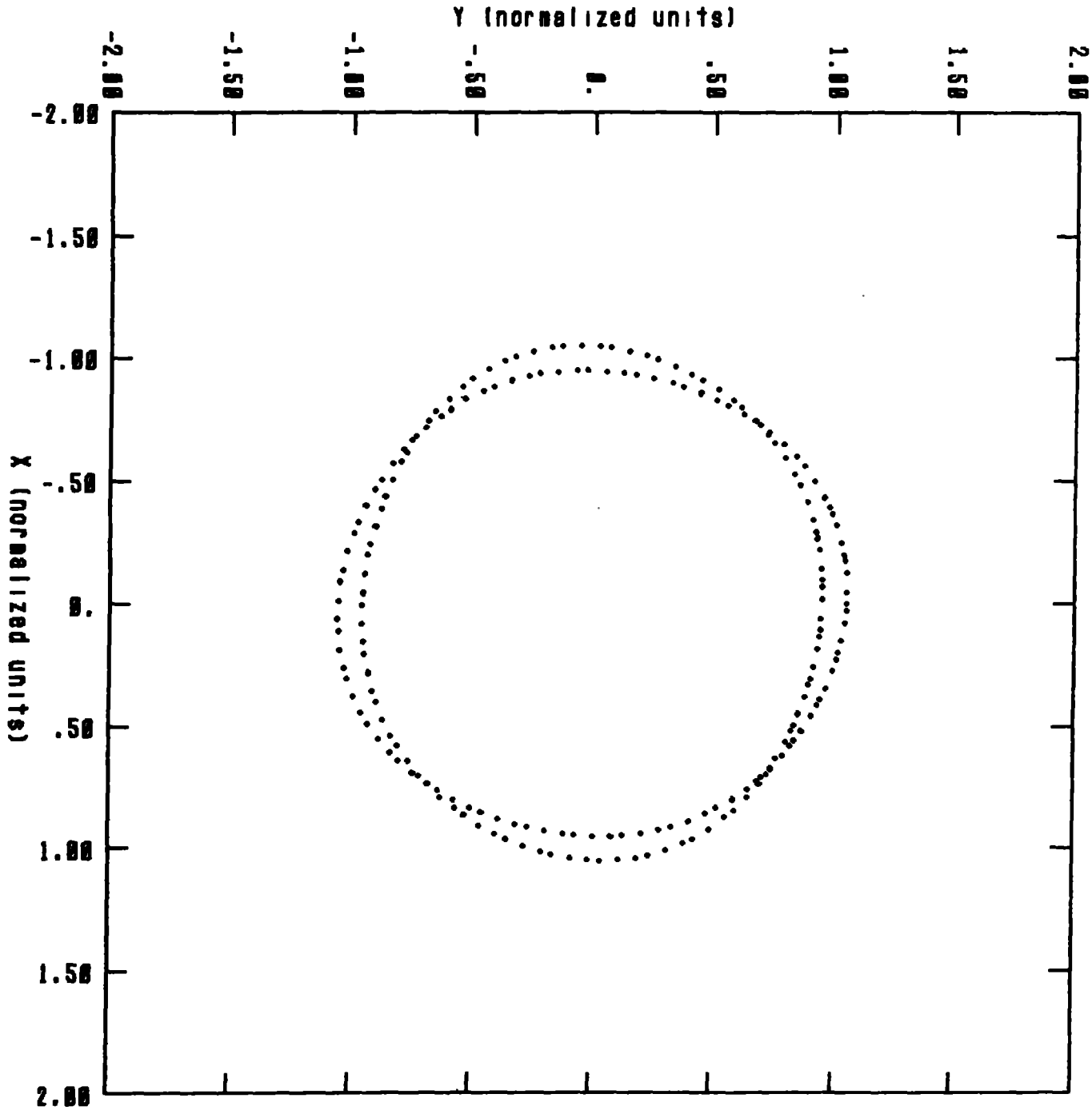
$$\phi_c = T_e \ln (n_p/n_c)$$

R. F. Post - Fig. 1

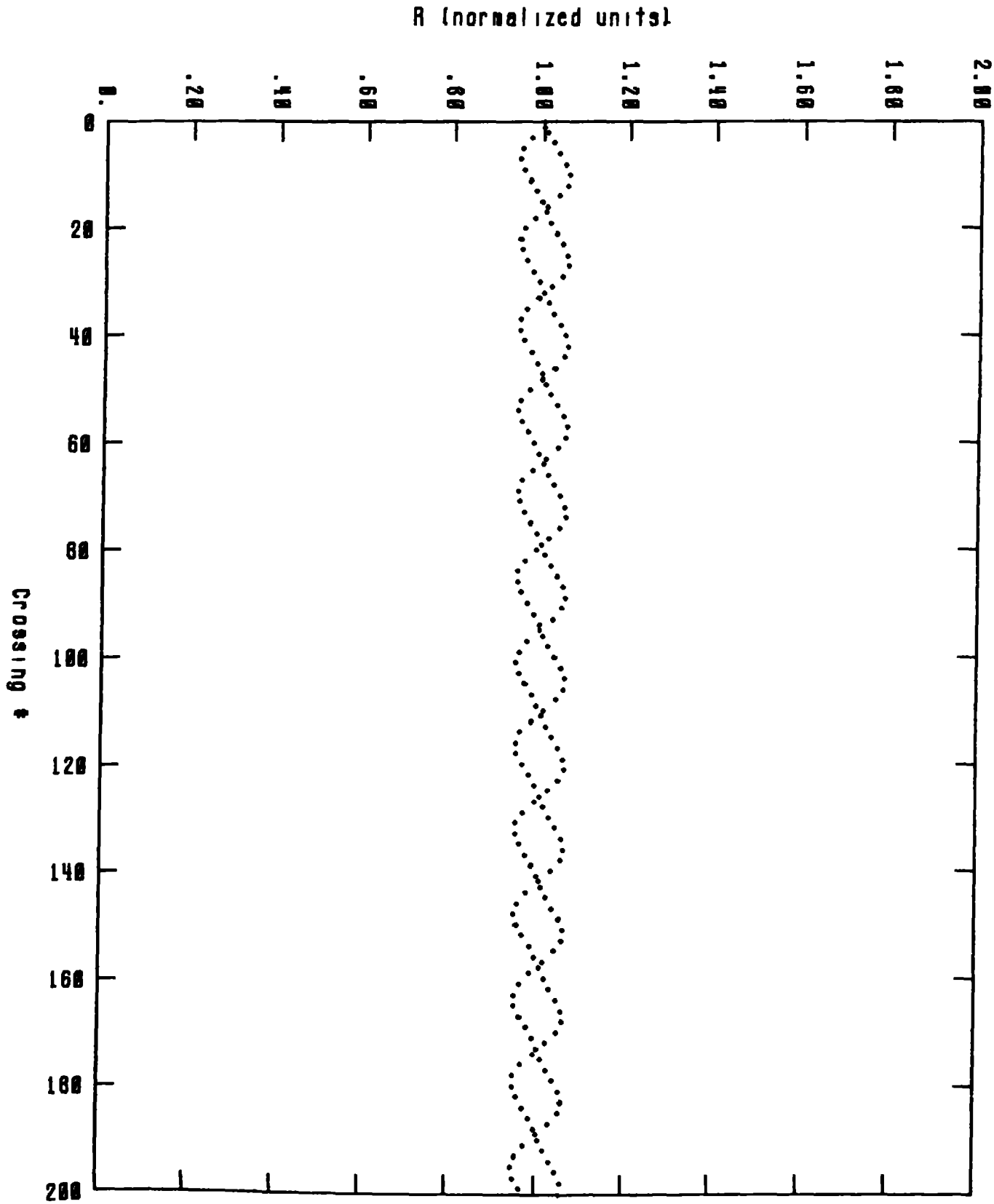
R. F. Post - Fig. 2


1.0
0.8
0.6
0.4
0.2
0.0
$\beta_\parallel$
STABLE
Infinite medium
Case I
Case II
0.0
0.2
0.4
0.6
0.8
1.0
$\beta_\perp$

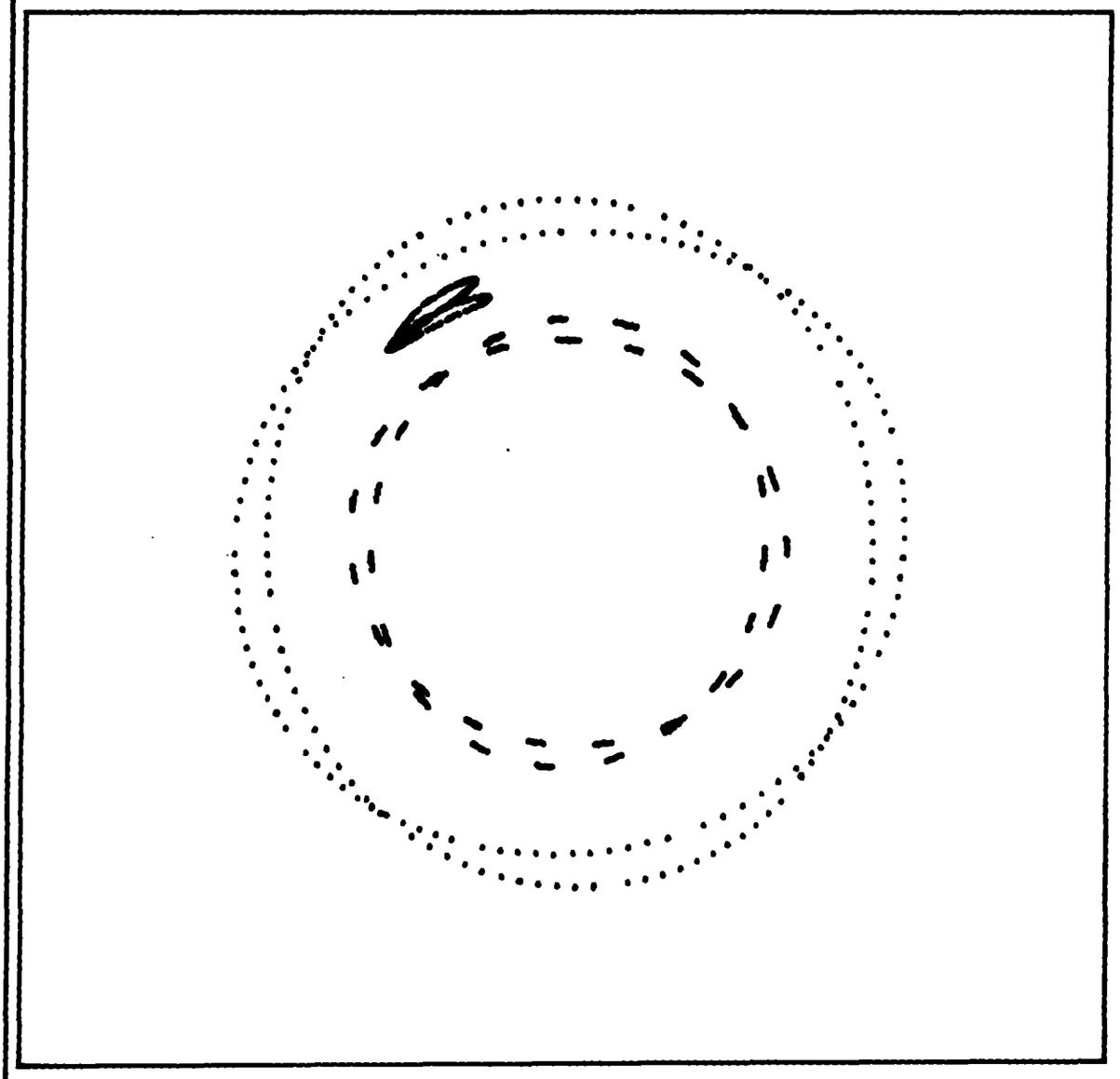
R. F. Post - Fig. 3

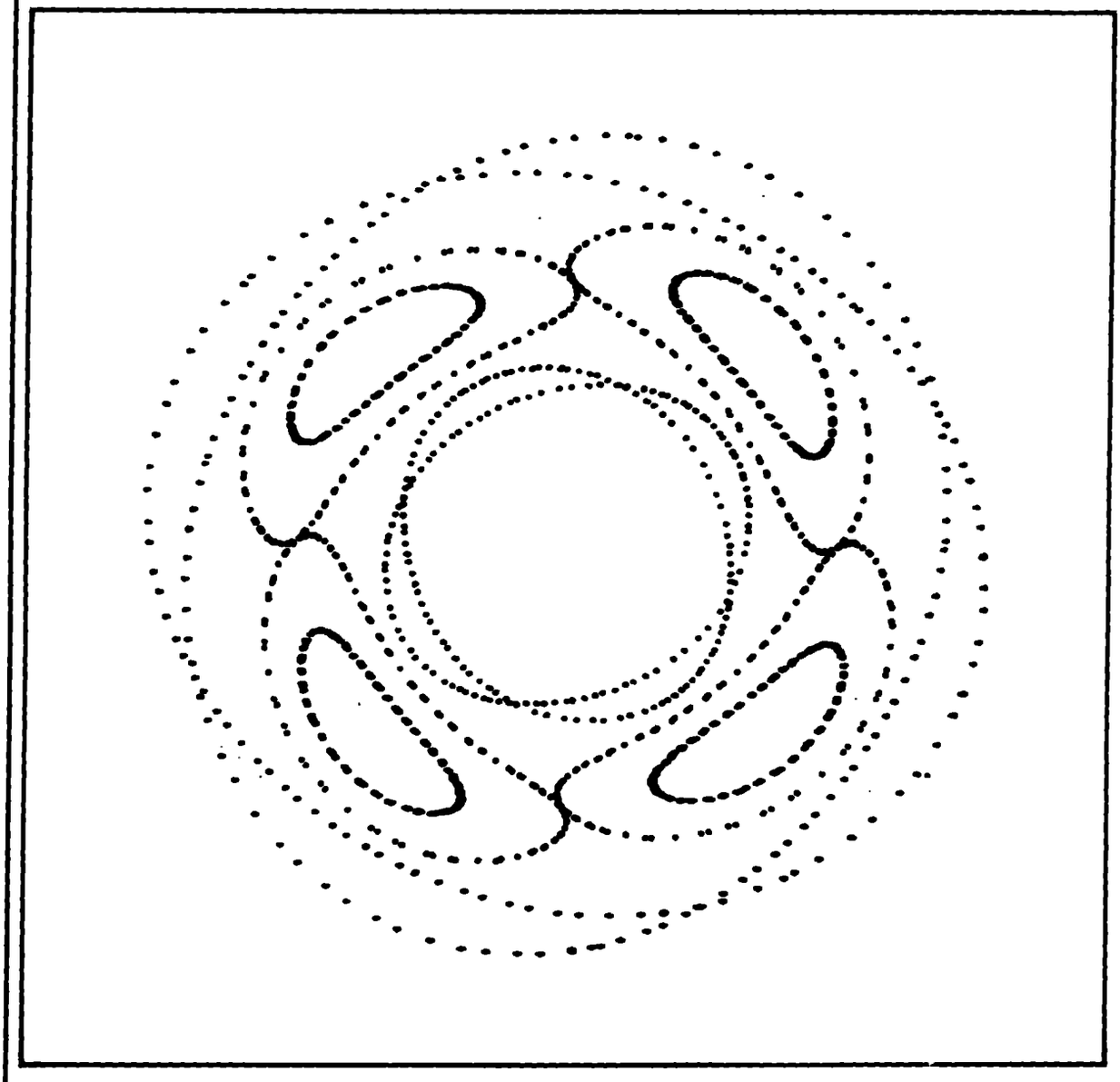

Y (normalized units)
-2.00
-1.50
-1.00
-.50
0.
.50
1.00
1.50
2.00
X (normalized units)
-2.00
-1.50
-1.00
-.50
0.
.50
1.00
1.50
2.00

R. F. Post - Fig. 4

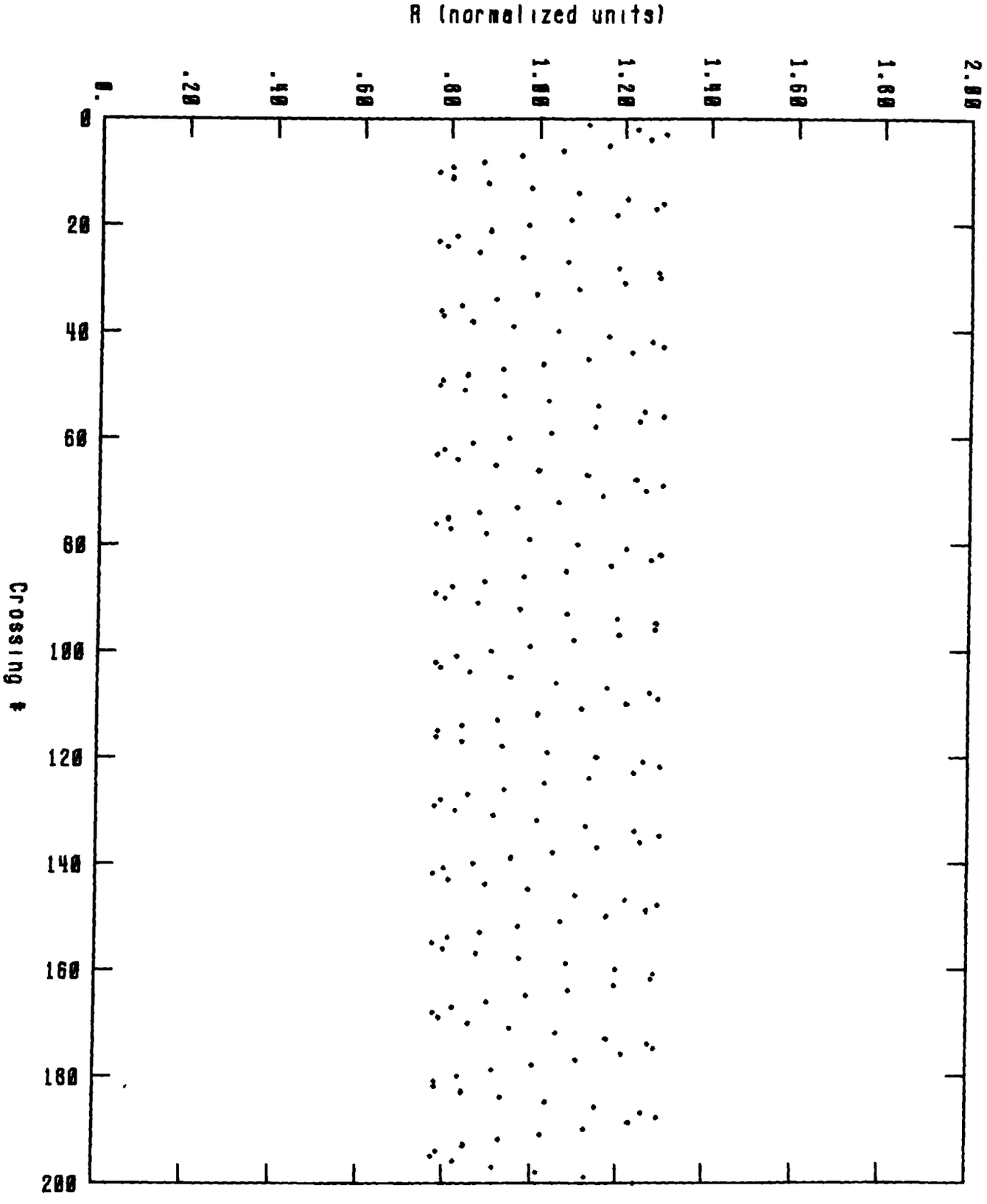
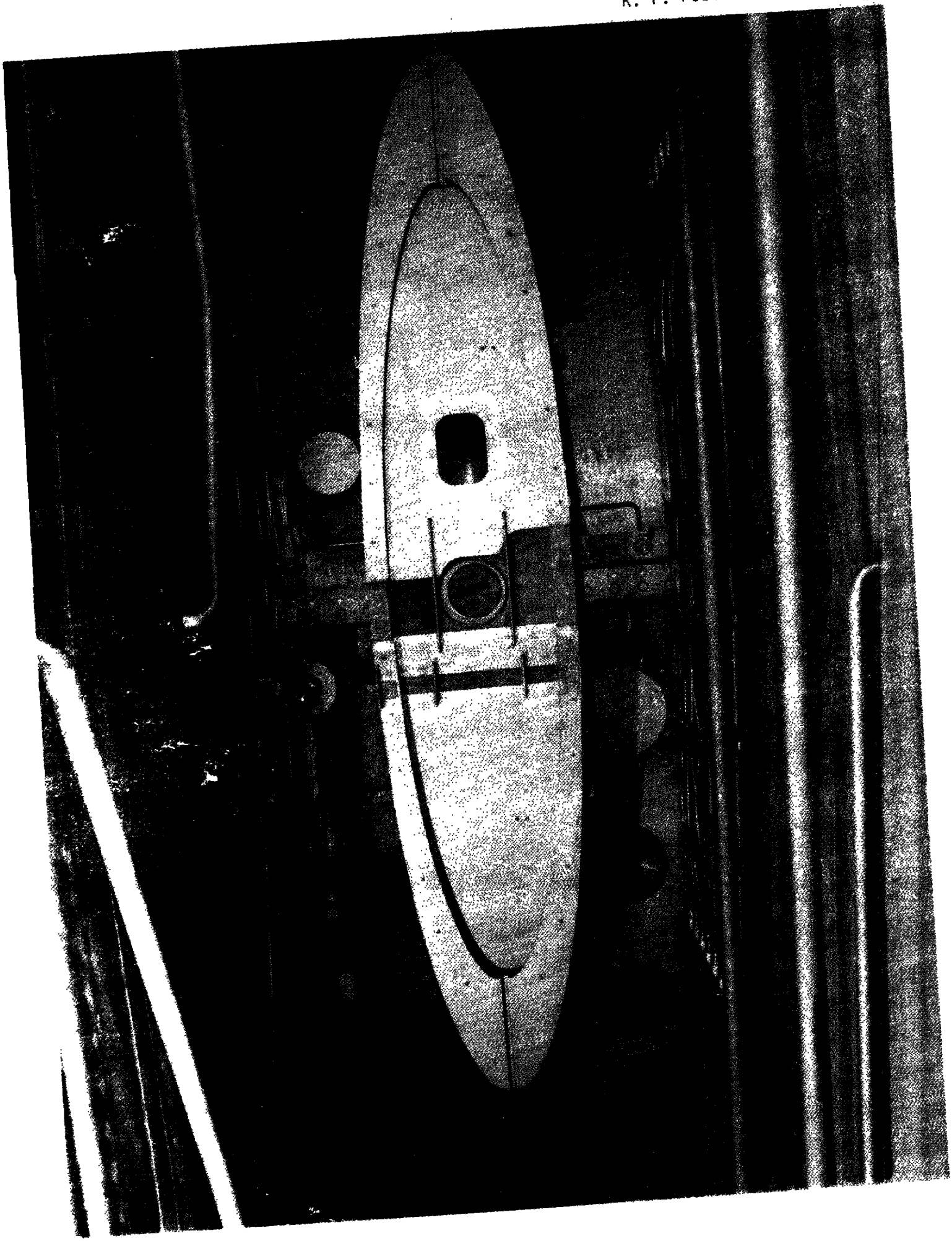

2.00
1.80
1.60
1.40
1.20
1.00
.80
.60
.40
.20
0.
R (normalized units)
0
20
40
60
80
100
120
140
160
180
200
Crossing #

R. F. Post - Fig. 5





R. F. Post - Fig. 7


R (normalized units)
0.
.20
.40
.60
.80
1.00
1.20
1.40
1.60
1.80
2.00
Crossing #
0
20
40
60
80
100
120
140
160
180
200